Assemblable Interlocking Polyominoes

HELLO!

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Interested in Computer Graphics, Computational Geometry and Algorithms and Theory

Currently working with professor **Christoph Hoffmann**

Outline

Brief background about Polyominoes

Finding Assemblable Interlocking Polyominoes

Current results

Working ideas

Brief background about Polyominoes

Let’s start with the basic stuff

“A polyomino is a plane geometric figure formed by joining one or more equal squares edge to edge.”

Polyomino: https://en.wikipedia.org/wiki/Polyomino

Some history

First problem published in 1907 by **Henry Ernest Dudeney**.

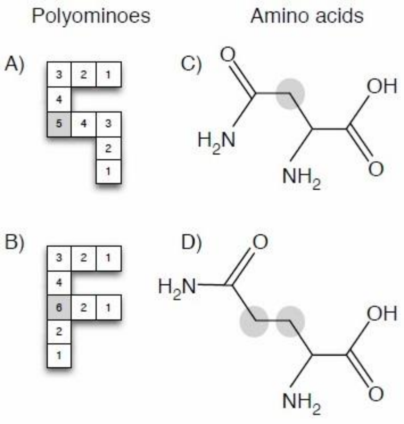
Solomon Golomb named them in 1953.

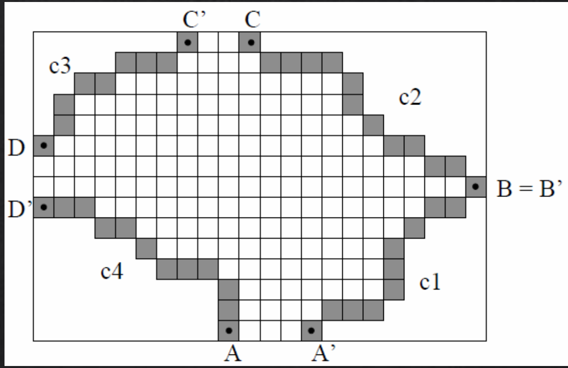
Martin Gardner popularized them in **Scientific American**.

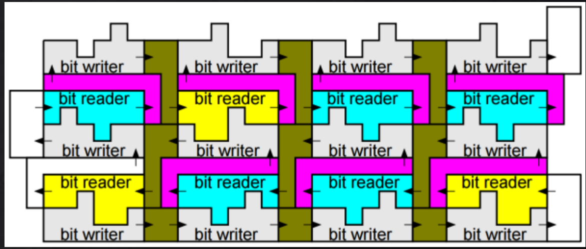
Now everyone is talking about them... Well, everyone **interested**.

Golomb, Solomon. (1965) Polyominoes. Charles Scribner's Sons, New York.

Some polyominoes in real life (aka not video games)







S. E. Ahnert, I. G. Johnston, T. M. A. Fink, J. P. K. Doye, A. A. Louis, Self-assembly, modularity and physical complexity, 17 December 2009, 10.1103/PhysRevE.82.026117, http://arxiv.org/abs/0912.3464
Isabelle Debled-Rennesson, Jean-Luc Rémy, Jocelyne Rouyer-Degli, Detection of the discrete convexity of polyominoes, Discrete Applied Mathematics, Volume 125, Issue 1, 15 January 2003, Pages 115-133, ISSN 0166-218X, http://dx.doi.org/10.1016/S0166-218X(02)00227-5.
Sándor P. Fekete, Jacob Hendricks, Matthew J. Patitz, Trent A. Rogers, Robert T. Schweller, Universal Computation with Arbitrary Polyomino Tiles in Non-Cooperative Self-Assembly, 14 August 2014, arXiv:1408.3351, http://arxiv.org/abs/1408.3351#

Tetris: Best polyomino application known so far

Thanks to Alexey Pajitnov.

One of the most popular videogames in history.

Computationally speaking, its complexity is NP-Complete.

Ron Breukelaar, Erik D. Demaine, Susan Hohenberger, Hendrik Jan Hoogeboom, Walter A. Kosters, and David Liben-Nowell, "Tetris is Hard, Even to Approximate", International Journal of Computational Geometry and Applications, volume 14, number 1-2, 2004, pages 41-68.

PENTRIS

Have you played it?

Do you think it is easier than Tetris?

Good luck!

Traditional research with/using polyominoes

Plane Tessellation

Puzzles commonly ask for tiling a given region with a given set of polyominoes, such as the 12 pentominoes.

Counting Polyominoes

No formula has been found except for special classes of polyominoes. A number of estimates are known, and there are algorithms for calculating them.

Discrete Tomographies

Useful for the reconstruction of two dimensional objects from their two orthogonal projections.

Protein Folding (poly GP map)

The mix of a genotype and a self-assembly process on a square lattice leads to the formation of phenotypes with different square tile building blocks conjoined along interacting edges.

Self-Assembly (polyTAM)

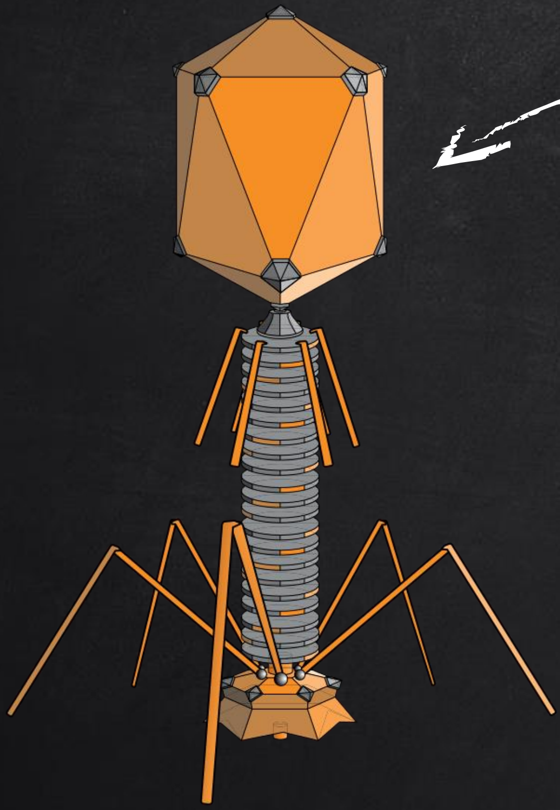
Polyominoes have enough geometric complexity to allow a polyTAM system at temperature 1, composed only of tiles of that shape, to perform Turing universal computation.

Interlocking Properties

A system of grid polygons such that the pieces interlock themselves (no piece can be moved far away from the rest).

Finding Assemblable Interlocking Polyominoes

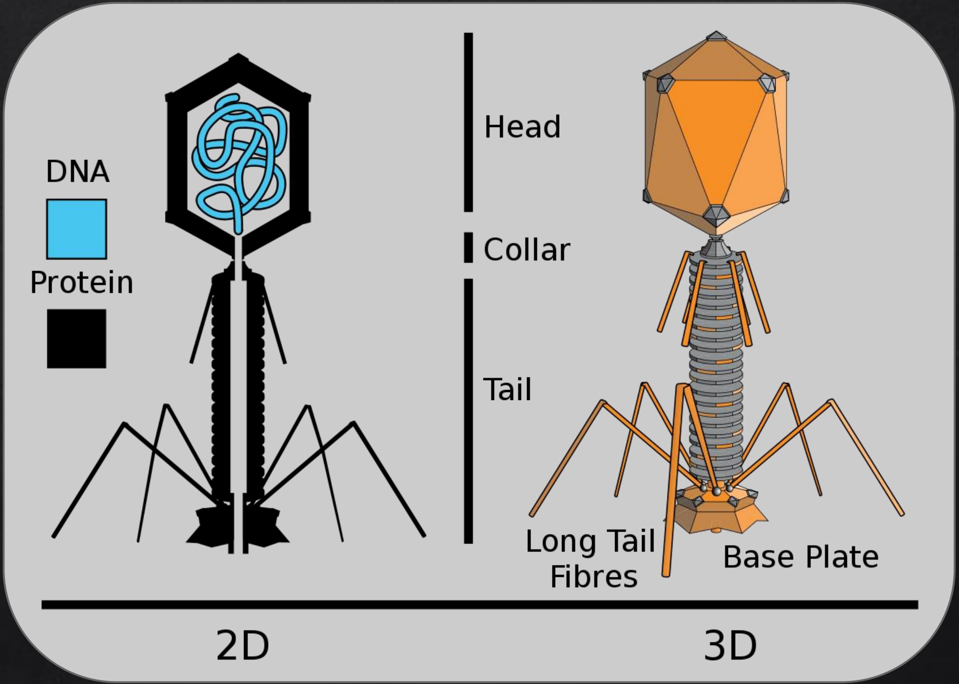
What you need to know in a nutshell

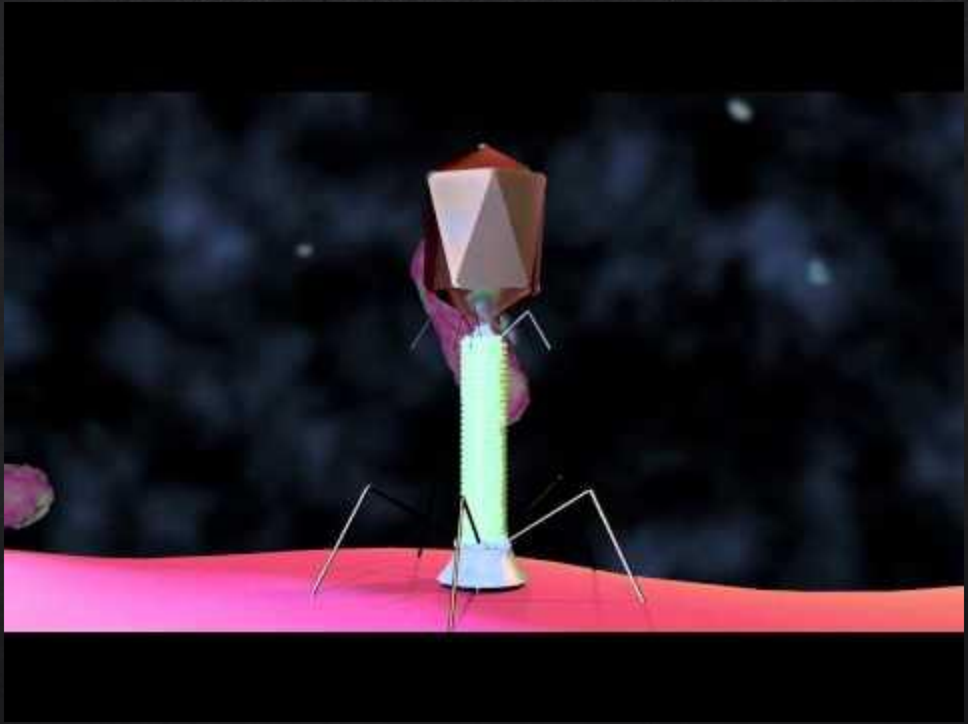


Do you know what's the structure of the left?

a) A self-deployed non-used lunar module prototype
b) A proposed antenna for long-distance communications
c) The structure of a myovirus bacteriophage
d) The next lovely robot for Star Wars Episode VIII

The structure of a typical myovirus bacteriophage



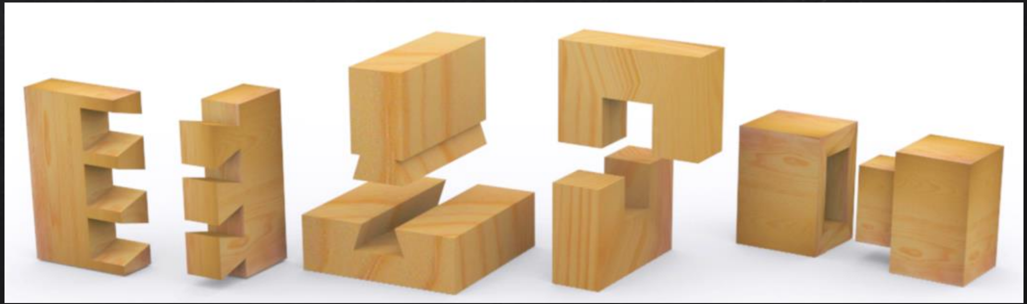


“So, the big problem that biologists and virologists are facing is to understand **how those building blocks get together** to form the container.”

Reidun Twarock – Department of Mathematics at the University of York.

Geometry: A new weapon in the fight against viruses. 20 May 2015. http://www.gresham.ac.uk/lectures-and-events/geometry-a-new-weapon-in-the-fight-against-viruses

Recent advance in furniture design

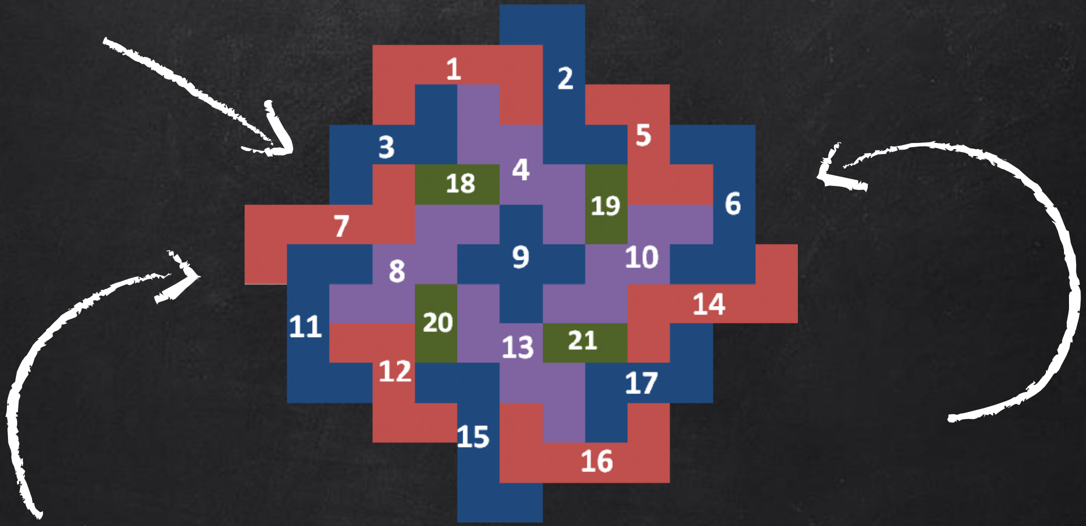


Chi-Wing Fu, Peng Song, Xiaoqi Yan, Lee Wei Yang, Pradeep Kumar Jayaraman, and Daniel Cohen-Or. 2015. Computational interlocking furniture assembly. ACM Trans. Graph. 34, 4, Article 91 (July 2015), 11 pages. DOI=http://dx.doi.org/10.1145/2766892

“

“A system of polyominoes is **collectively interlocked** if no polyominoes can be moved arbitrarily far away [in the plane] from any of the others.”

Interlocking Hexominoes. From: Dhawan, S., Abel, Z., Complexity of interlocking polyominoes, http://arxiv.org/pdf/1112.4087.pdf



An Interlocked Set

Skeptical? Let's see the demo!

Sidharth Dhawan, Zachary Abel. Complexity of Interlocking Polyominoes. 17 December 2011. arXiv:1112.4087. http://arxiv.org/abs/1112.4087

Not a simple task, it's just intractable

It is PSPACE Hard!

The Totally Quantified Boolean Formula (TQBF) problem, which is PSPACE Complete, can be reduced to the sliding blocks problem.

Determining interlockedness is PSPACE hard for a system of polyominoes with only hexominoes and smaller polyominoes.

What is PSPACE?

It is the class of all languages recognizable by polynomial space bounded Deterministic Turing Machine programs that halt on all inputs.

It is still unknown whether there exist problems solvable in polynomial space that cannot be solved in polynomial time. (is P = PSPACE?)

Is it always PSPACE Hard?

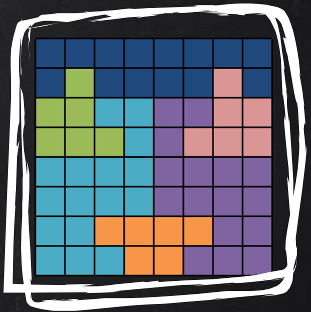
It is if we want to tessellate a complete region using hexominoes and smaller polyominoes.

But sometimes we don't want to interlock all the region…

Sidharth Dhawan, Zachary Abel. Complexity of Interlocking Polyominoes. 17 December 2011. arXiv:1112.4087. http://arxiv.org/abs/1112.4087

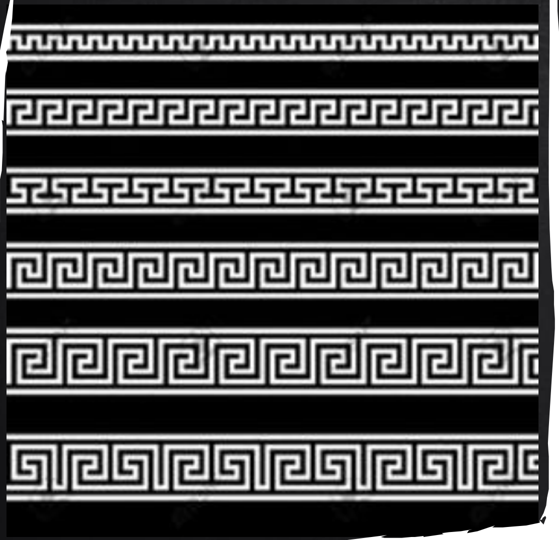
Lessons from our previous work

- Space filling curves
- Full tessellation of the lattice
- Key pieces are too big



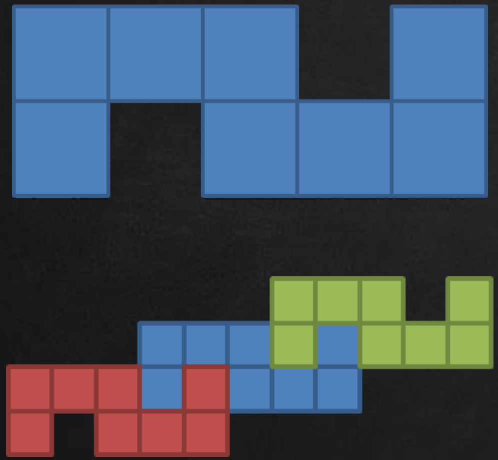
- We have to do better
- Maybe we do not require all the interlockings
- Still "assemblable"

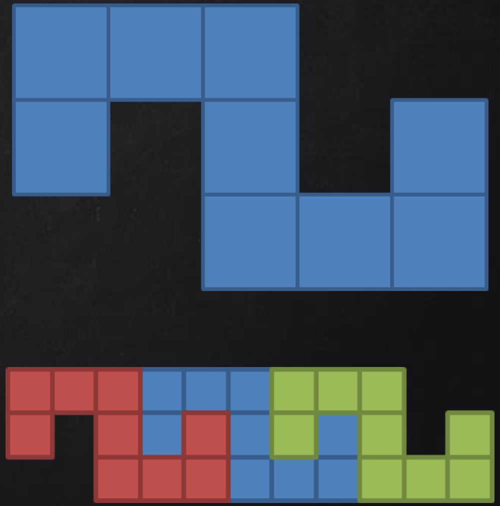
The greeks were right!

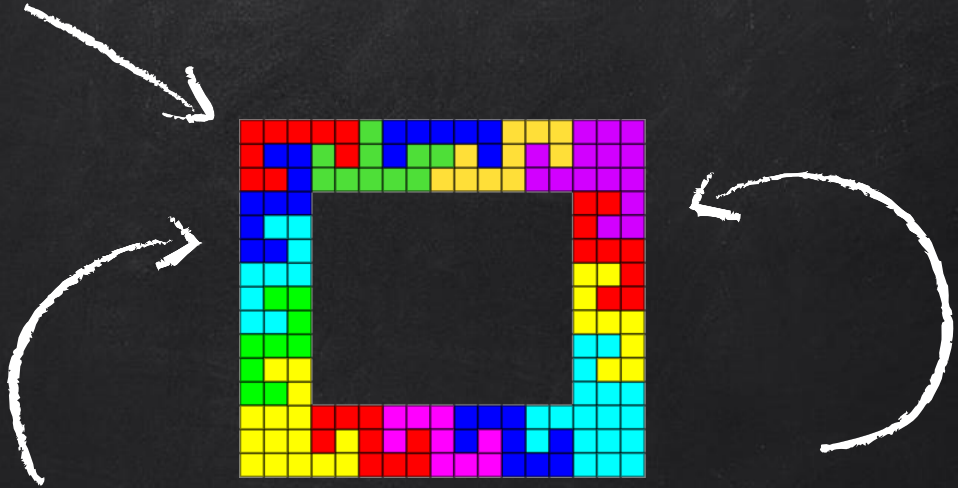


Left: https://www.pinterest.com/pin/427842651861322224/
Right: By English: Euphronios as potter (signed), Onesimos as painter - Jastrow (2006), Public Domain, https://commons.wikimedia.org/w/index.php?curid=1413335

Our approach: S-shaped pieces

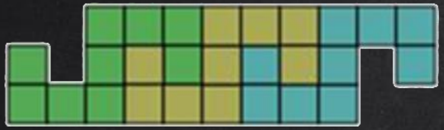


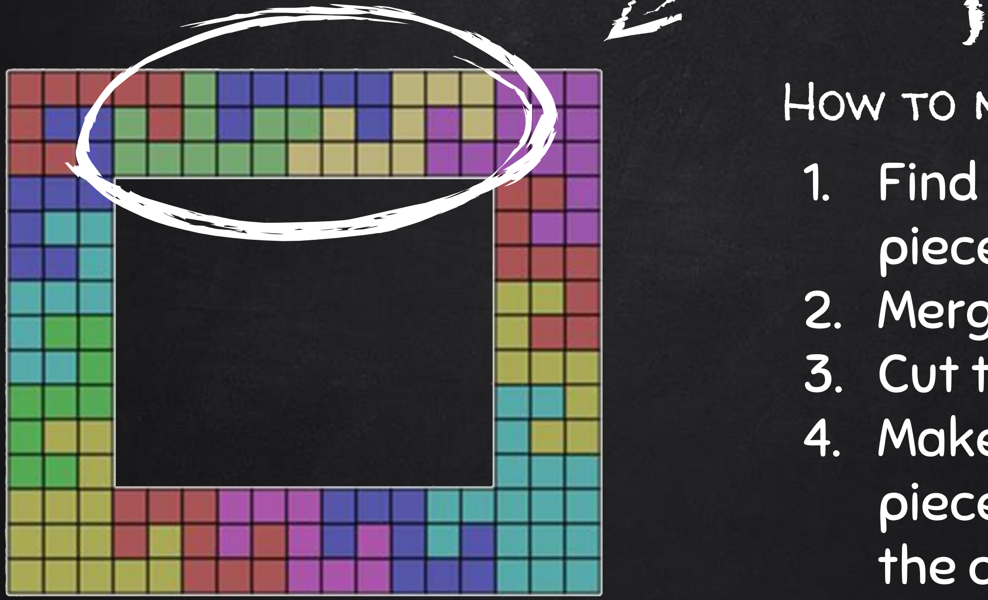




Assemblable Sets

Skeptical again? Let’s return back to the demo!



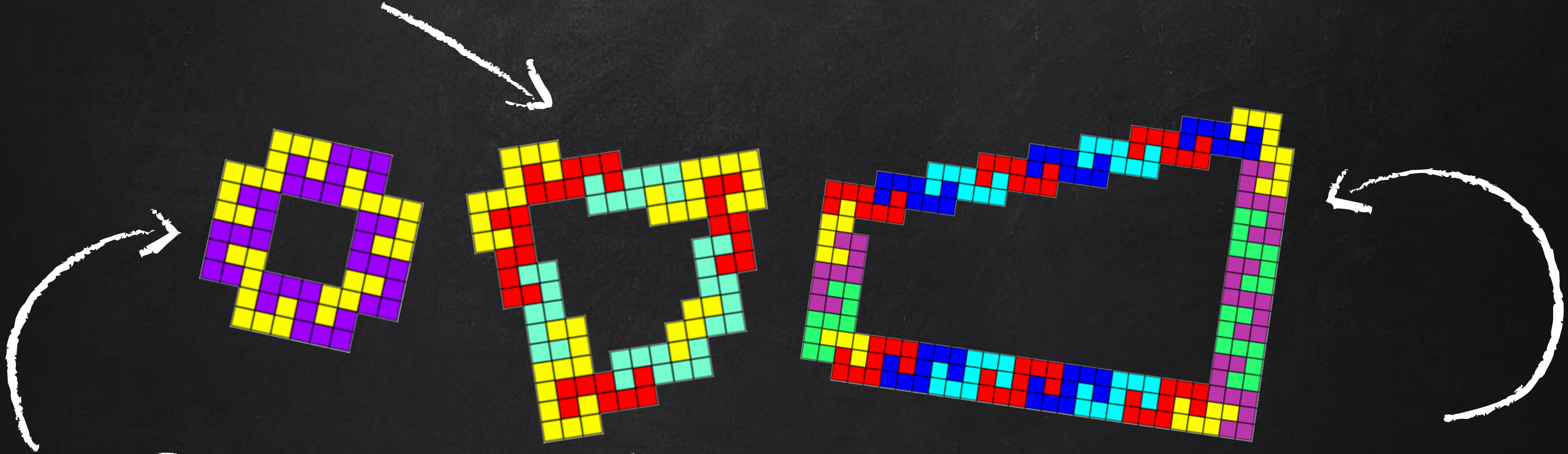


How to make it assemblable?

1. Find three consecutive s-shaped pieces
2. Merge them
3. Cut the blue piece (key piece!)
4. Make a step cut below the key piece following the direction of the original merged s-shaped pieces.

Current Results

We have some interesting sets to show



Clovers, Foxgloves and Staircases

No more doubts! The demo has the answer

Working Ideas

And some other tentative thoughts as well

Degrees of interlocking Polyominoes

Is it always PSPACE Hard?

It is when using hexominoes and smaller polyominoes.

Still not known how complex it is when using additional help (i.e., glueing) or some predefined pieces (i.e., s-shaped pieces).

Example: k-glues interlocking class

Let's say we allow a **k** number of glues between squares in the lattice.

- Boundaries? (i.e., min k?)
- Algorithm?
- Is the solution always an interlocking set?
- Running time?
- Can we do better/generalize?

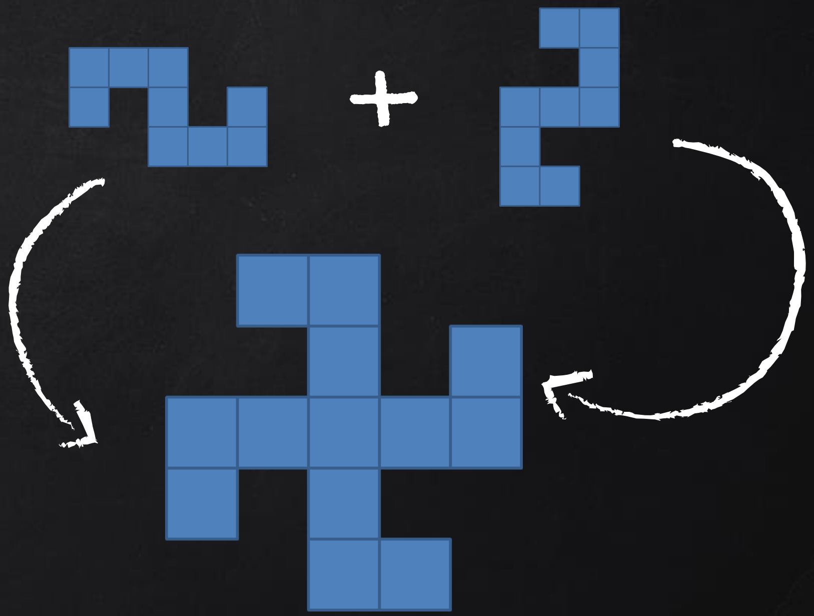
How about more general interlocking pieces?

Goal: Interlock everything!

S-shaped pieces are good for linear segments.

Idea: replicate the design in the four directions.

Not a popular polyomino though.



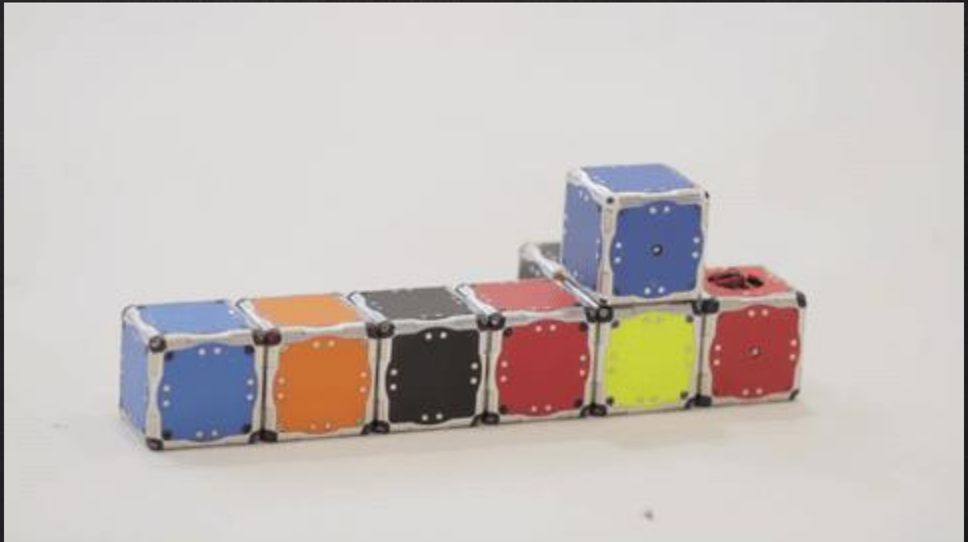
Crowdsourcing: Let humans find the answer

Playing video games for science: You get smarter, we get data.

A mobile game app for finding interlocking patterns designed by people.

Having a heuristic would be great!

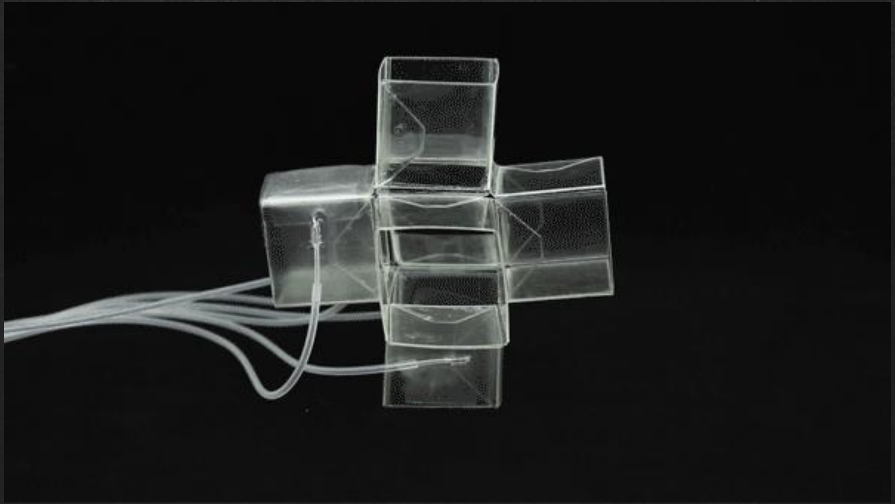
Games That Solve Real Problems: Crowdsourcing Biochemistry. (Oct 27, 2011) http://www.forbes.com/sites/techonomy/2011/10/27/games-that-solve-real-problems-crowdsourcing-biochemistry/#1146e6831199



Robotic Uses

Polyominoes + Robots + Self-Assembly

Small cubes that self-assemble (October 3, 2013) https://www.youtube.com/watch?v=6aZbJS6LZbs



Self-Interlocking

Foldings + Snapology = Interlocking 3D structures

3-D material changes shape as it prepares for next task. http://news.harvard.edu/gazette/story/2016/03/3-d-material-changes-shape-as-it-prepares-for-next-task/

THANKS!

Any questions?

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https://www.cs.purdue.edu/homes/abejara/

References

All 18 pentominoes
(https://commons.wikimedia.org/wiki/File%3AAll_18_Pentominoes.svg)
By R. A. Nonenmacher (Created by me) [GFDL
(http://www.gnu.org/copyleft/fdl.html) or CC BY-SA 4.0-3.0-2.5-2.0-1.0
(http://creativecommons.org/licenses/by-sa/4.0-3.0-2.5-2.0-1.0)], via
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