Dealing with Shape, Simulation and Equilibrium of Convex Interlocking Assemblies

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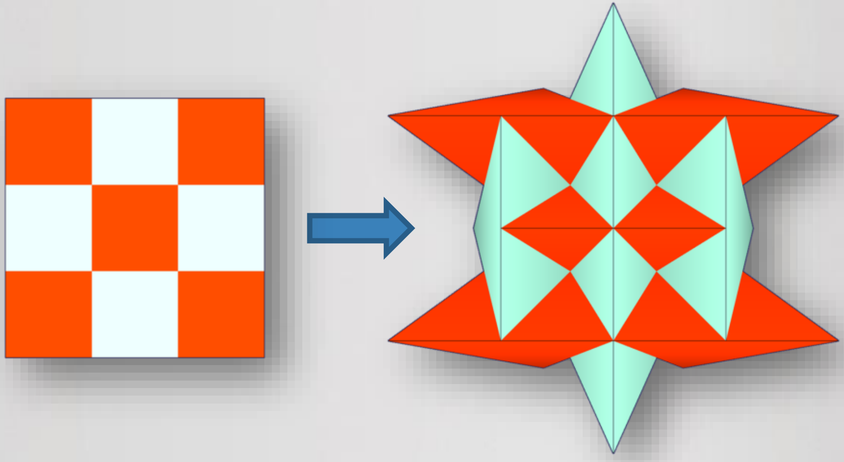
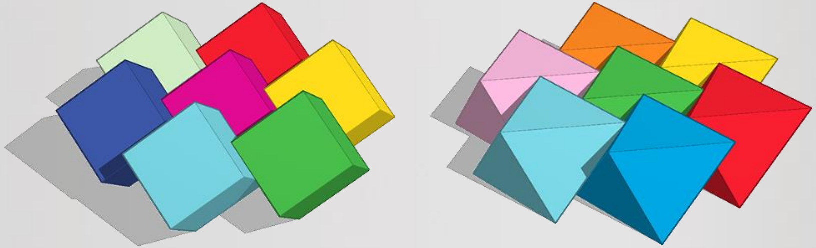
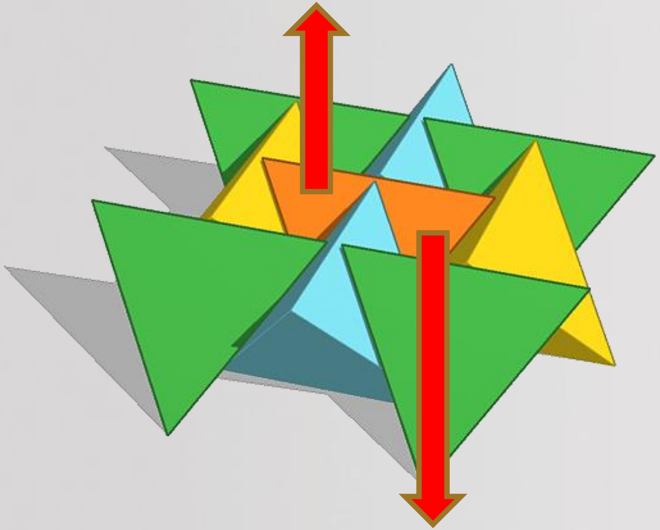
CONVEX INTERLOCKING

“Blocks of **special shape** arranged such that each piece is kept in place by **kinematic constrains** imposed through the shape and **mutual arrangement** of the elements.”

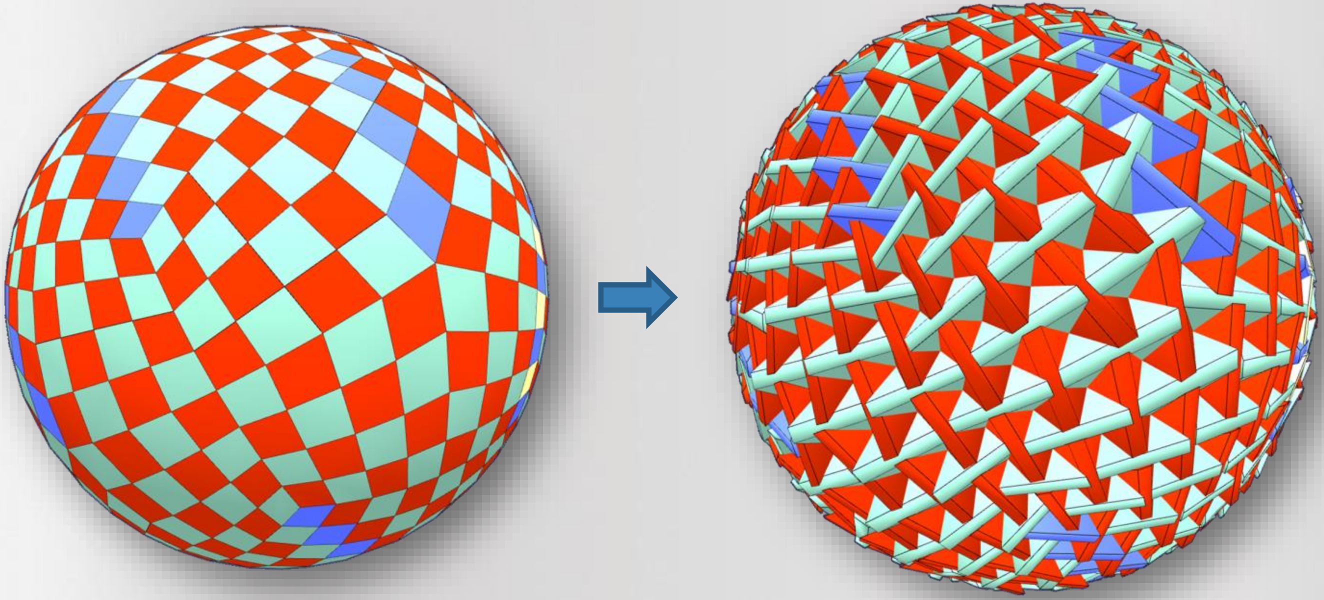




Pulling action constrained by blue pieces



Pushing action constrained by yellow pieces



Goal: Given a mesh, generate a self-supporting structure using a convex interlocking assembly

OUR PROCESS IS AS FOLLOWS

Shape

From a face to a convex polyhedron

Simulation

Check if it collapses

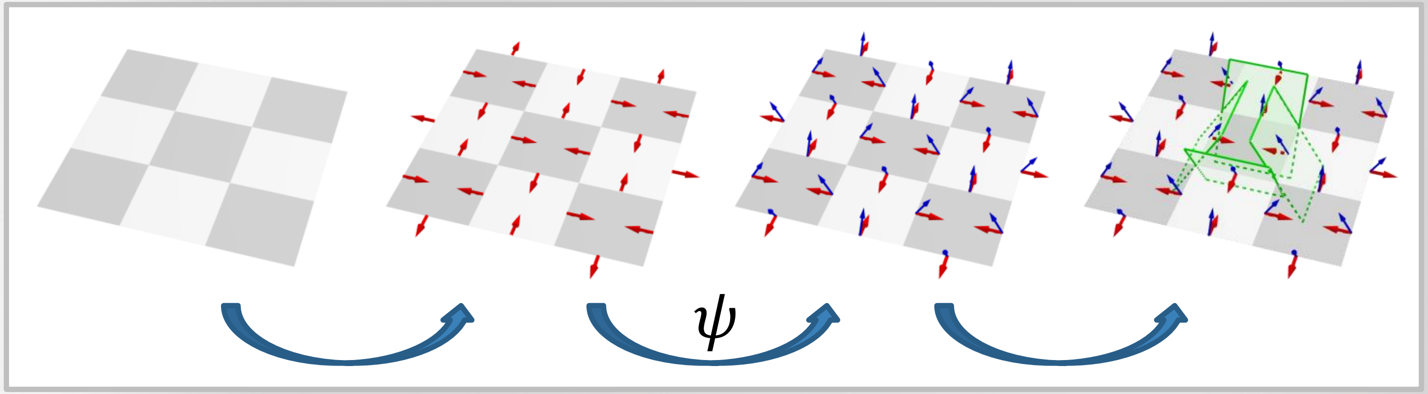
Equilibrium

Inquire why it collapses

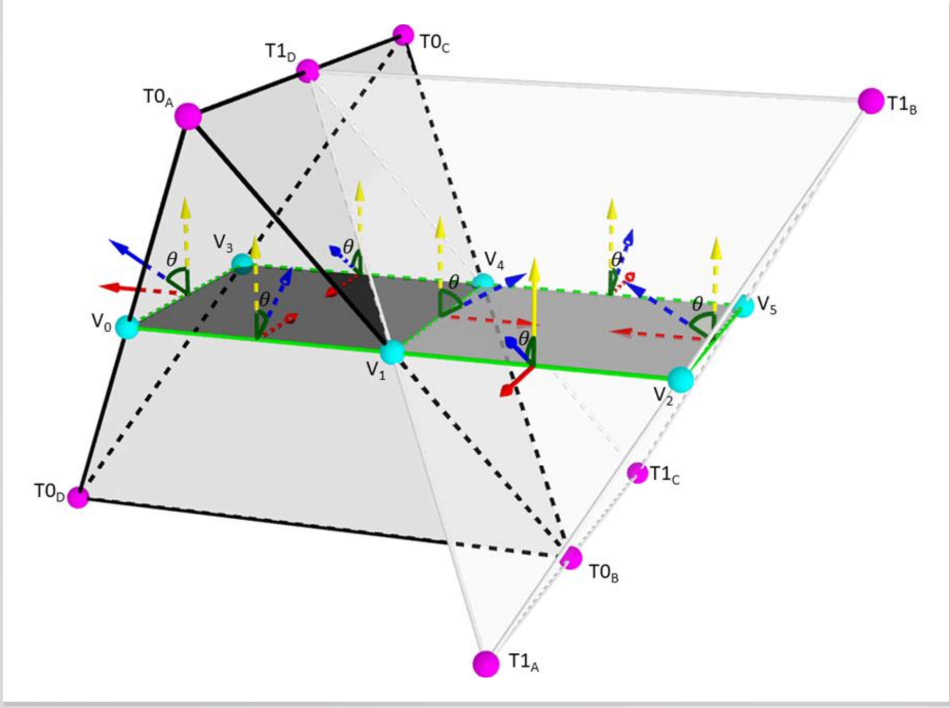
1.
SHAPE

We need some building blocks to begin with

OVERALL APPROACH

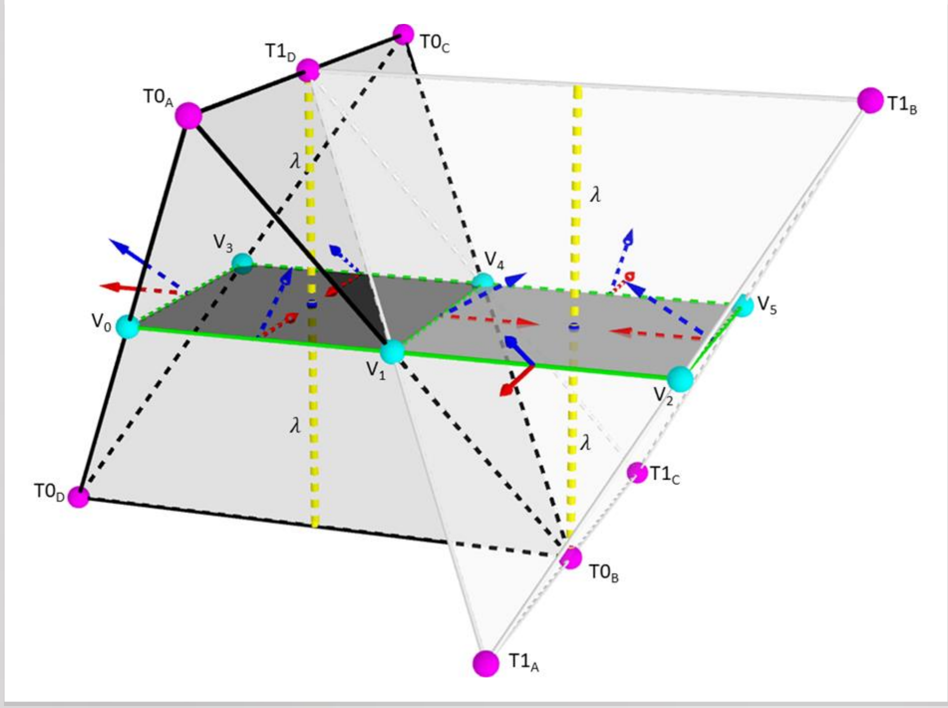


This is what people do: use ***angles***

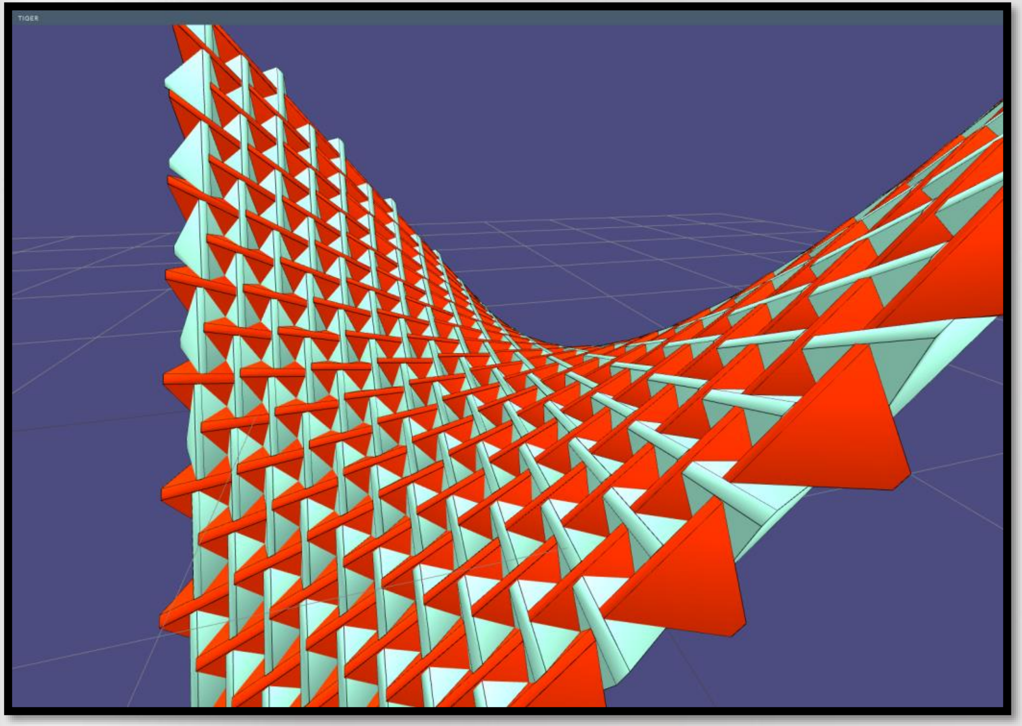


TILTING ANGLE METHOD

*This is what we propose: use **heights***



HEIGHT-BISECTION METHOD

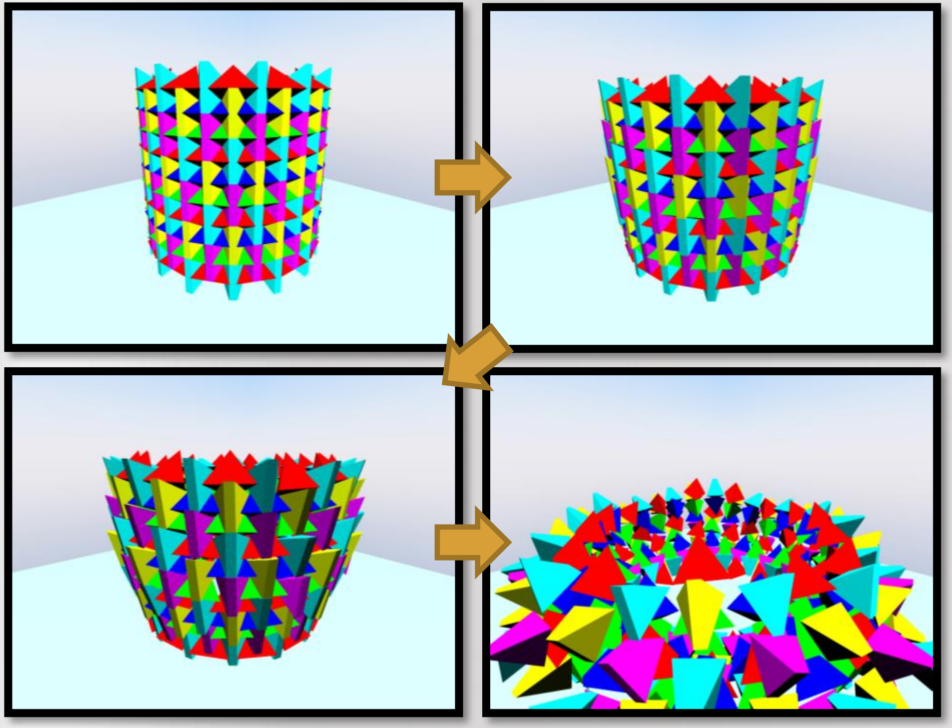


Both methods available on TIGER

Quick demo!

2.
SIMULATION

Because simulating is less “expensive” than 3D printing



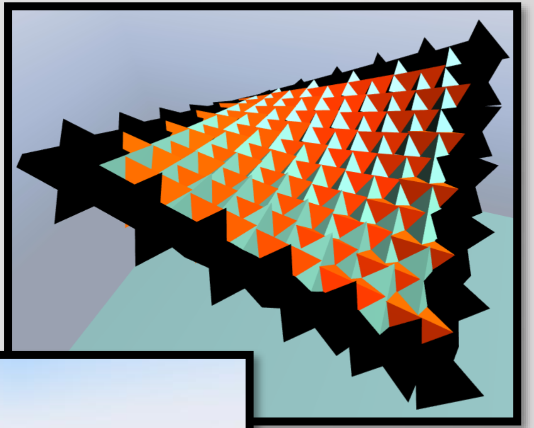
Assemblies prevent pull and push actions on a piece along the direction of the normal vector of the respective face.

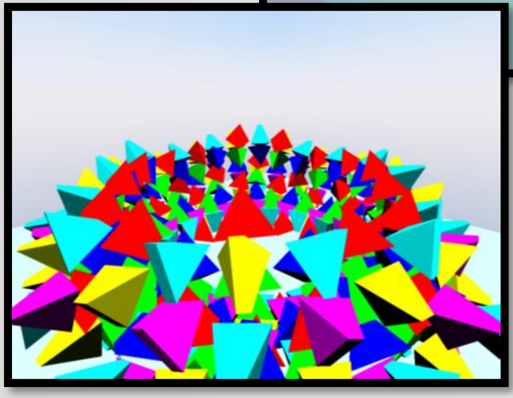
Force of gravity pulls all pieces downwards.

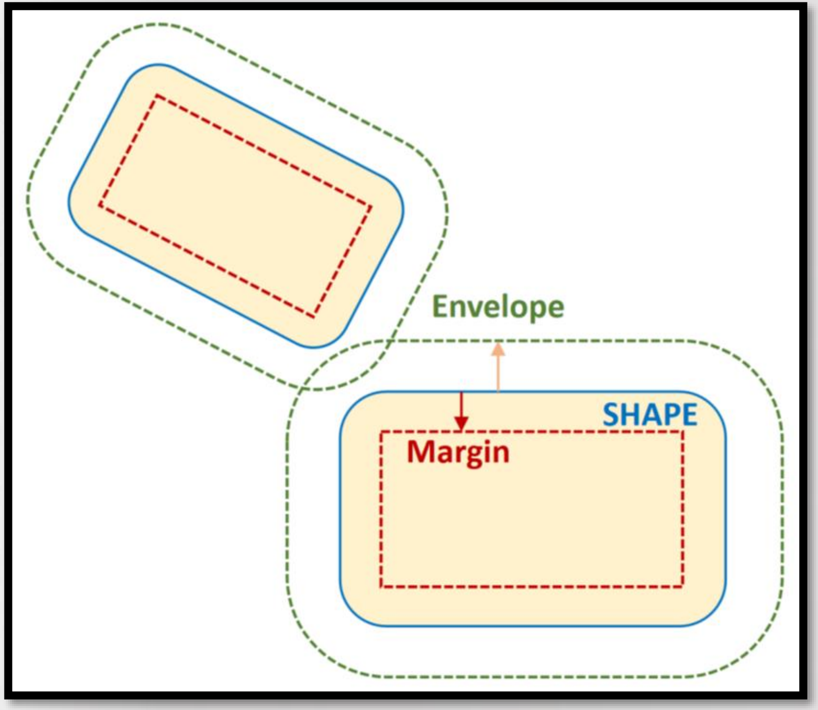
Support structure is expected to mitigate the action of gravity.

Assumptions and variables considered on physical simulations:

- Force of gravity
- Static friction
- Rigid body geometry
- Equal density for each piece
- Support frame fixed in space
- Tolerance?







There are two tolerances to deal with: envelope and margin.

Envelope: *Search volume.*

Shape: *Actual geometry.*

Margin: *Range of penetration.*

Choose wisely; otherwise, be prepared for numerical instability

3.
EQUILIBRIUM

Check why the structure collapses and fix it

“MEASURING” EQUILIBRIUM… ?

FEA

Finite Element Analysis checks **material failures** and current state of the **stress**.

Manual adjustments required.

TNA

Thrust Network Analysis considers **axial forces** at block interfaces.

Requires a shell structure that can be projected onto a 2D plane.

Infeasibility

Determines how **far** a structure is from a **stable configuration**.

Suppports arbitrary topologies.

We choose this approach!

MEASURE OF INFEASIBILITY

We follow this approach for analysis the structure equilibrium and reach a stable configuration.

Reference:

*Whiting, Emily, Hijung Shin, Robert Wang, John Ochsendorf, and Frédo Durand. 2012. "**Structural Optimization of 3D Masonry Buildings.**" ACM Trans. Graph. 31 (6): 159:1–159:11. https://doi.org/10.1145/2366145.2366178.*

Procedural Modeling of Structurally-Sound Masonry Buildings

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Figure 1: *Our method generates models of masonry buildings that are structurally stable. In this building based on Cluny Abbey in France, parameters controlling the flying buttresses, columns, and window sizes have been automatically optimized to support a stone vaulted ceiling. The right image shows reaction forces at the ground plane. We solve for forces at all block interfaces, and apply a compression-only constraint for masonry materials.*

Abstract

We introduce structural feasibility into procedural modeling of buildings. This allows for more realistic structural models that can be interacted with in physical simulations. While existing structural analysis tools focus heavily on providing an analysis of the stress state, our proposed method automatically tunes a set of designated free parameters to obtain forms that are structurally sound.

Keywords: procedural modeling, statics, structural stability, architecture, optimization, physics

1 Introduction

Content creation for virtual environments has become a bottleneck in computer graphics and interactive applications. Geometric models are required to have high visual realism and also be suitable for use in physical simulations. Structurally stable models enhance realism in virtual environments by allowing characters to interact with the built surroundings, whereas models which are not consistent with mechanics might collapse under their own weight.

Procedural modeling has emerged as a powerful technique for generating architectural geometry. However, existing techniques focus on visual realism and do not account for the structural validity of the results. Users may not have intuition about the mechanics that govern structural stability, or knowledge of traditional proportions used in building design. Determining the precise dimensions of a structure that guarantee stability can be a tedious task. We present a method to automatically "snap" to feasible dimensions, while leaving control in the designer's hands for deciding which aspects of the model are variable.

Our contribution is to introduce physical constraints into procedural modeling methods. We solve an inverse statics problem: given a set of physical constraints and a building topology, we determine an appropriate shape. The user provides a set of production rules that describes the desired architectural style, along with a small set of free parameters. The relationship between rule parameters and internal forces in the structure is nonlinear. Using gradient-based nonlinear optimization, our method searches over the parameter space for a stable configuration.

We focus on masonry structures, which encompass historic cathedrals, stone bridges, brick walls, unreinforced concrete dams, and other common structures. Masonry constructions behave as undeformable rigid blocks with interaction forces limited to compression and friction [Heyman 1995]. In order to impose structural feasibility, a forward analysis tool is required to assess the soundness of a structure. However, current engineering tools based on finite element methods and elasticity theory [Zienkiewicz 1971] are not appropriate in this context because they focus on material failure and stress, and because the high stiffness of stone results in poorly conditioned numerical systems. In contrast, the critical factor in masonry structures is the geometric configuration and whether it is in static equilibrium. In particular, Block et al. [2006] demonstrated that linear elastic theory was unable to differentiate between a feasible masonry arch and an infeasible arch. For this reason, we revisit an approach introduced by Livesley [1978], and we present a new forward structural analysis method based on optimization under linear constraints.

ACM Reference Format

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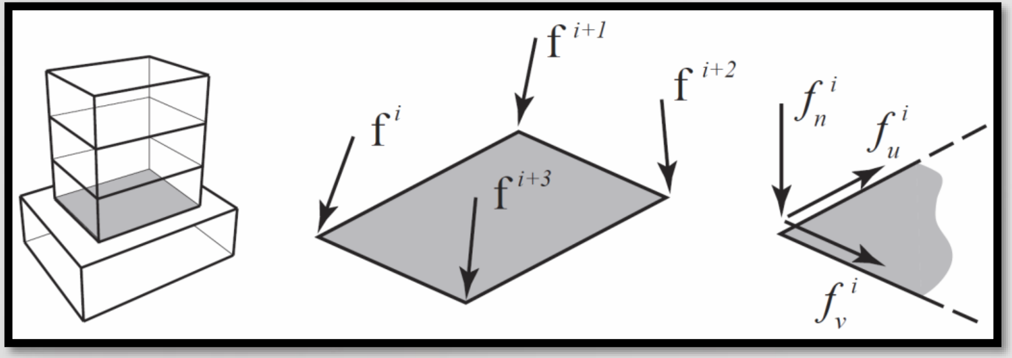


Equilibrium constraints for each block:

$$\mathbf{A}_{eq} \cdot \mathbf{f} + \mathbf{w} = 0$$

Compression constraints:

$$f_n^i \geq 0, \forall i \in \text{interface vertices}$$

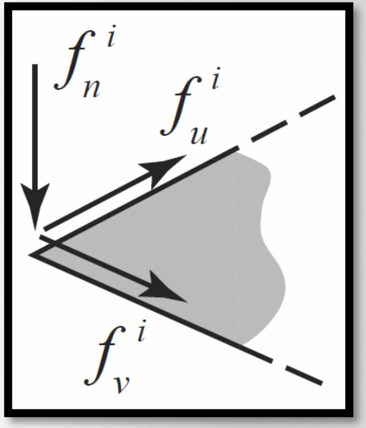
Friction constraints:

$$|f_u^i|, |f_v^i| \leq \alpha f_n^i, \forall i \in \text{interface vertices}$$

Structure is in **equilibrium** if a force solution **f** exists that satisfies such linear constraints.

Result is a yes/no answer.

FEASIBILITY (ALLOW SOME "GLUE")



Soften compression constraint by allowing a tensile force (**but penalize it!**)

$$f_n^i = f_n^{i+} - f_n^{i-}$$

$$f_n^{i+}, f_n^{i-} \geq 0$$

Penalty formulation:

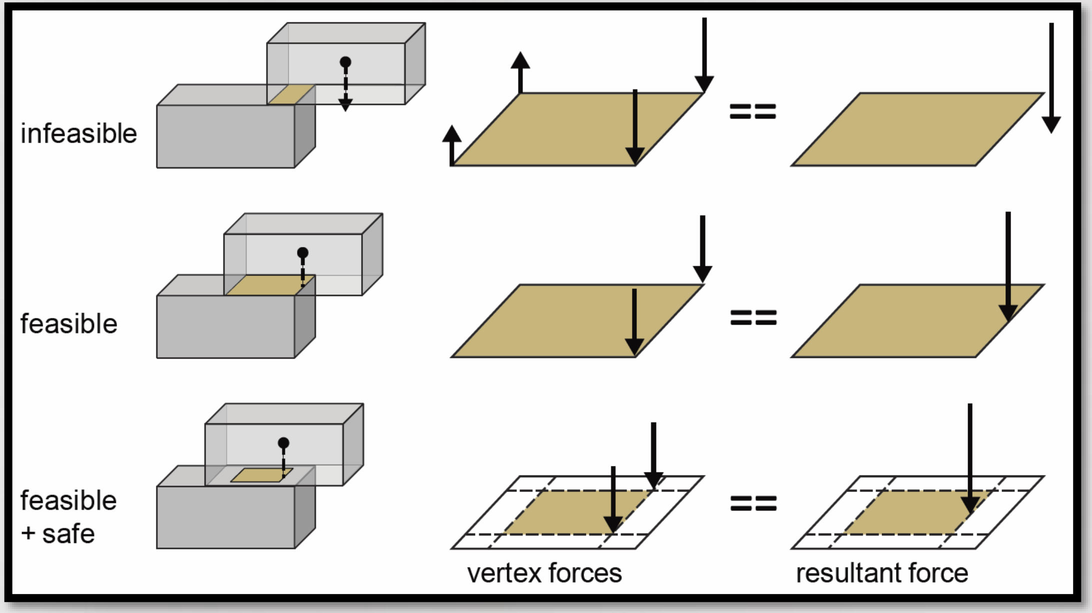
$$y(\psi) = \min_{\mathbf{f}} \sum_{i=0}^{n} \left(f_n^{i-}\right)^2$$

Such that:

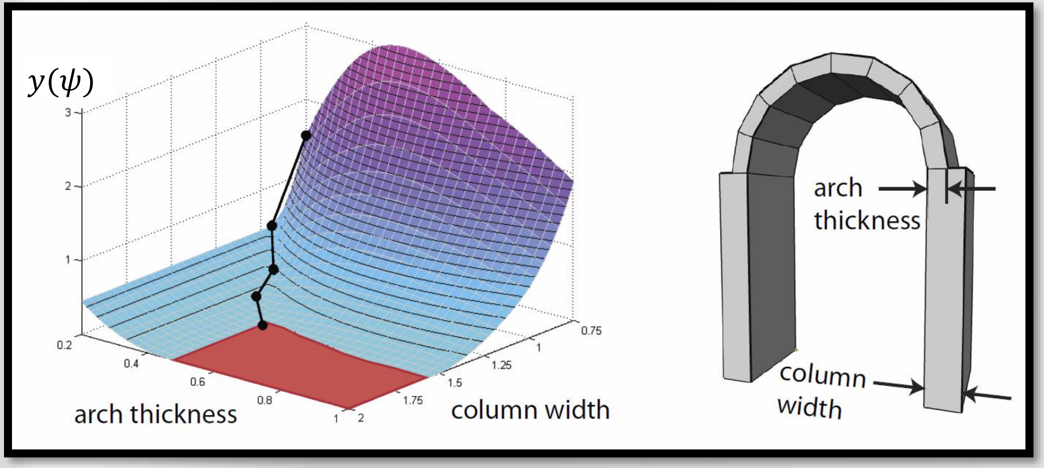
$$\mathbf{A}_{eq} \cdot \mathbf{f} = -\mathbf{W}$$

$$\mathbf{A}_{fr} \cdot \mathbf{f} \leq 0$$

$$f_n^{i+}, f_n^{i-} \geq 0 \; \forall i$$



Infeasible vs Feasible (plus safe kerns)



Example from reference paper. Here, ψ is the set of parameters (arch thickness and column width) for a parametric arch supported by columns.

Reshape parameters ψ by traversing $y(\psi)$:

$$\text{argmin}_{\psi}\ y(\psi)$$
$$lb \leq \psi \leq ub$$

A.K.A. Follow the gradient!

TL;DR

Allow tensile forces (glue)

It softness the compression constraint. We penalize the glue though.

Reshape blocks using the gradient

Use the gradient to move to a stable configuration. Repeat until you reach it (or something close to it).

CONCLUSIONS

Shape

Generate a valid assembly is possible using heights

Simulation

Watch out simulation parameters

Equilibrium

Calculate infeasibility and repeat

THANKS!

Any questions?

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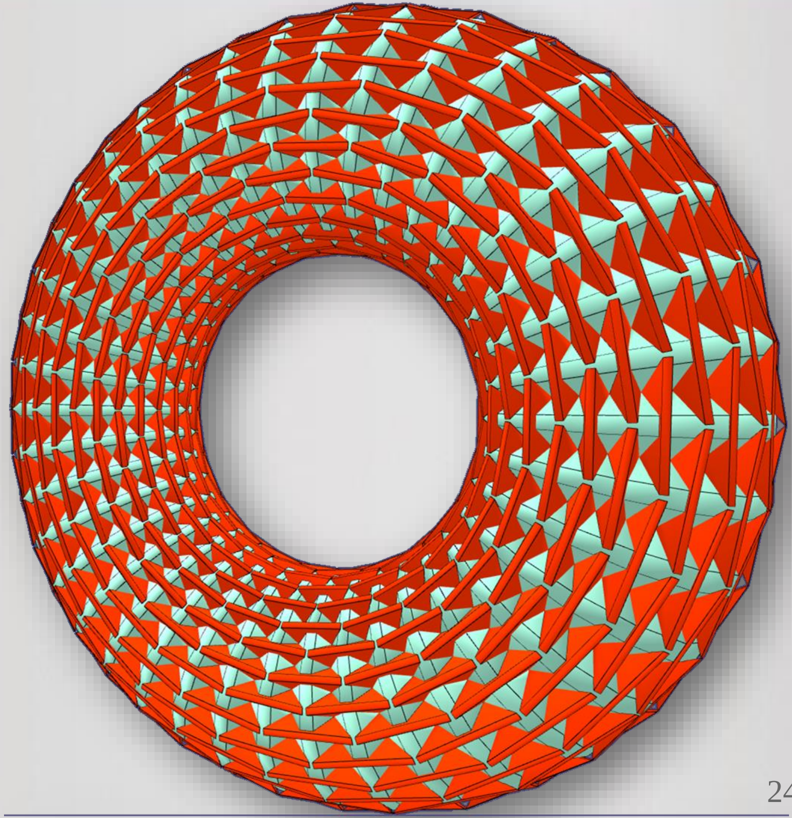

CREDITS

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