Topological Interlocking Cylinder Configurations: A Geometric Approach

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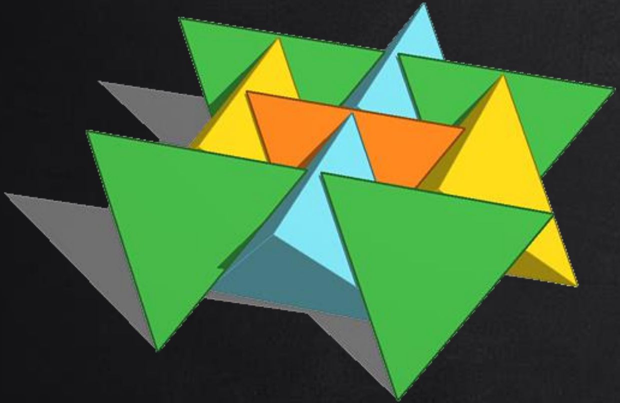
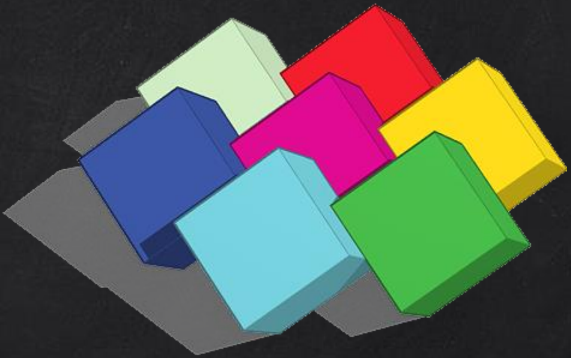
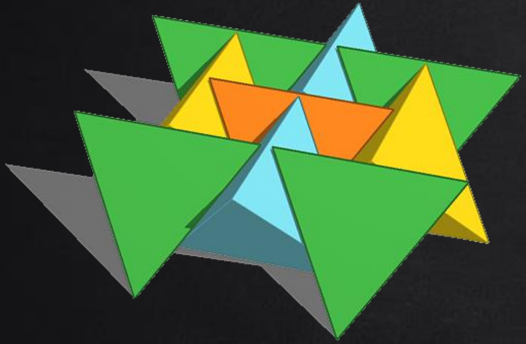




These are Topological Interlocking Configurations (TICs)

A TIC generation method that runs on any surface

A valid TIC should not have overlapping between pieces



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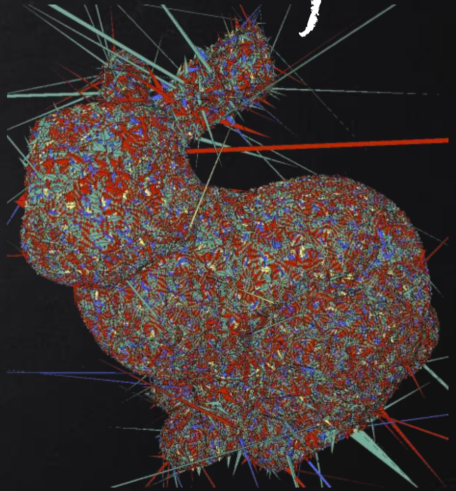


We want to generalize the concept for any kind of surface.

TICs as material

For a planar configuration using tetrahedra:

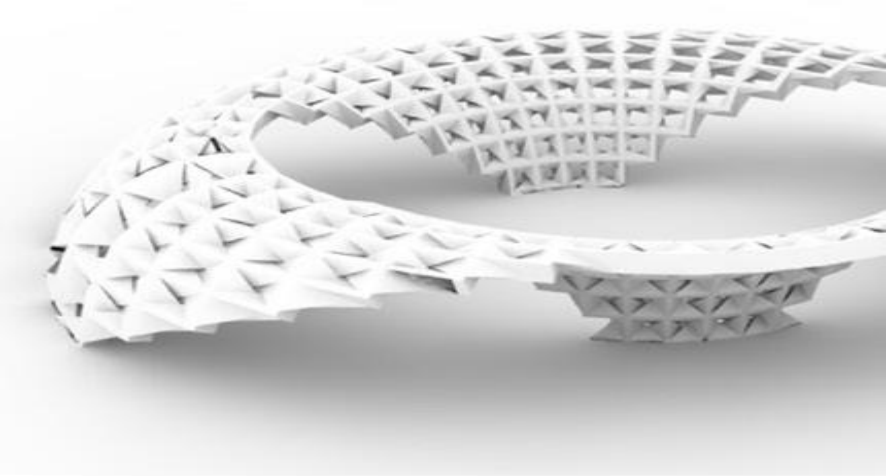
- Avoids crack propagation.
- Percolation threshold up to 59%
- Easy to manufacture.
- And many more!

Image: Dyskin *et al.* (2002)

TICs as Design

For a configuration based on any surface:

- "Complex" generation process.
- Peripheral Constraint.
- Pieces need help (e.g., glue or mortar).
- Geometric domain affects the shape of the pieces.



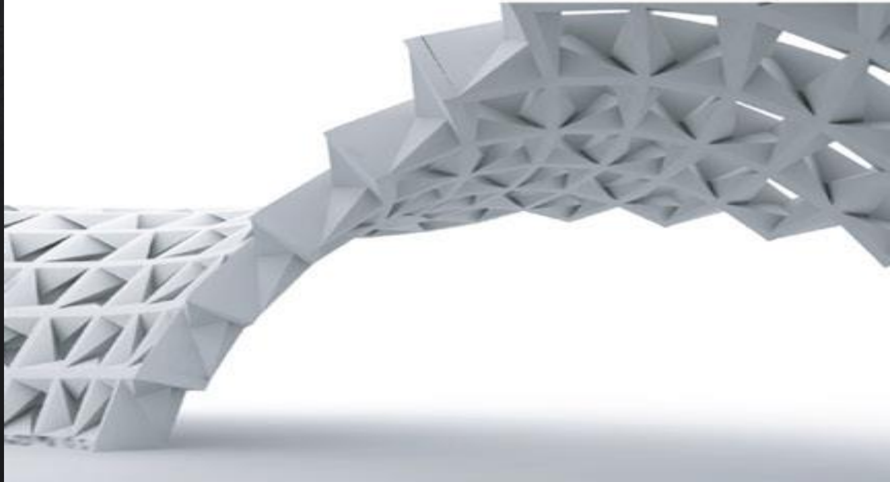
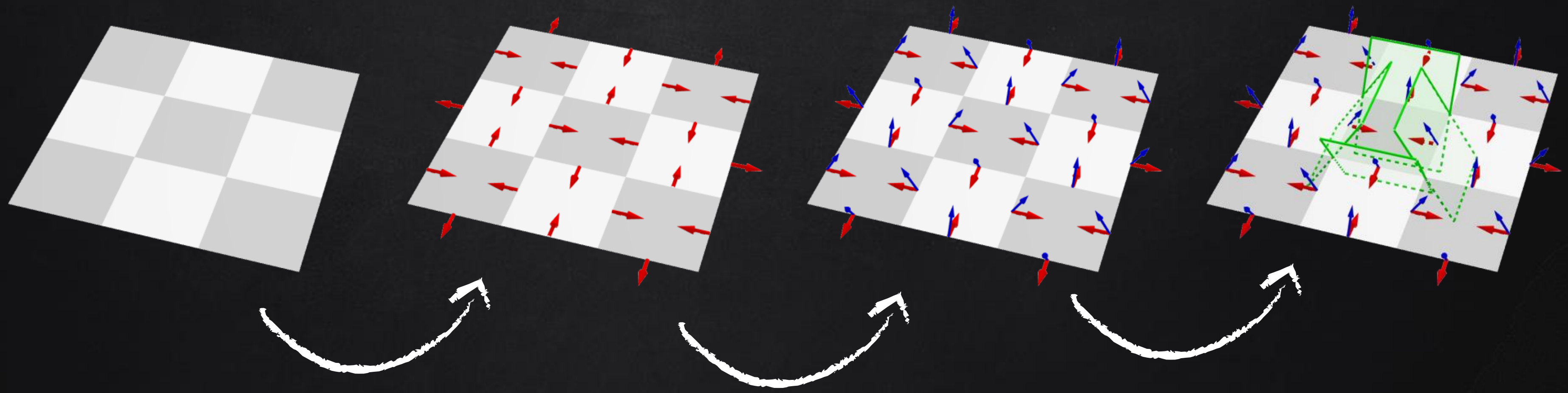


Image: Tessmann (2002)

How to make a TIC?
(Traditional Method)



Chessboard

Map a chessboard* on the geometric domain (aka surface) and generate the pieces!

* In theory, any tessellation composed of even-sided tiles would work.
Image from: General Framework for Discrete Surface Ricci Flow. Zhang et al. (2014)

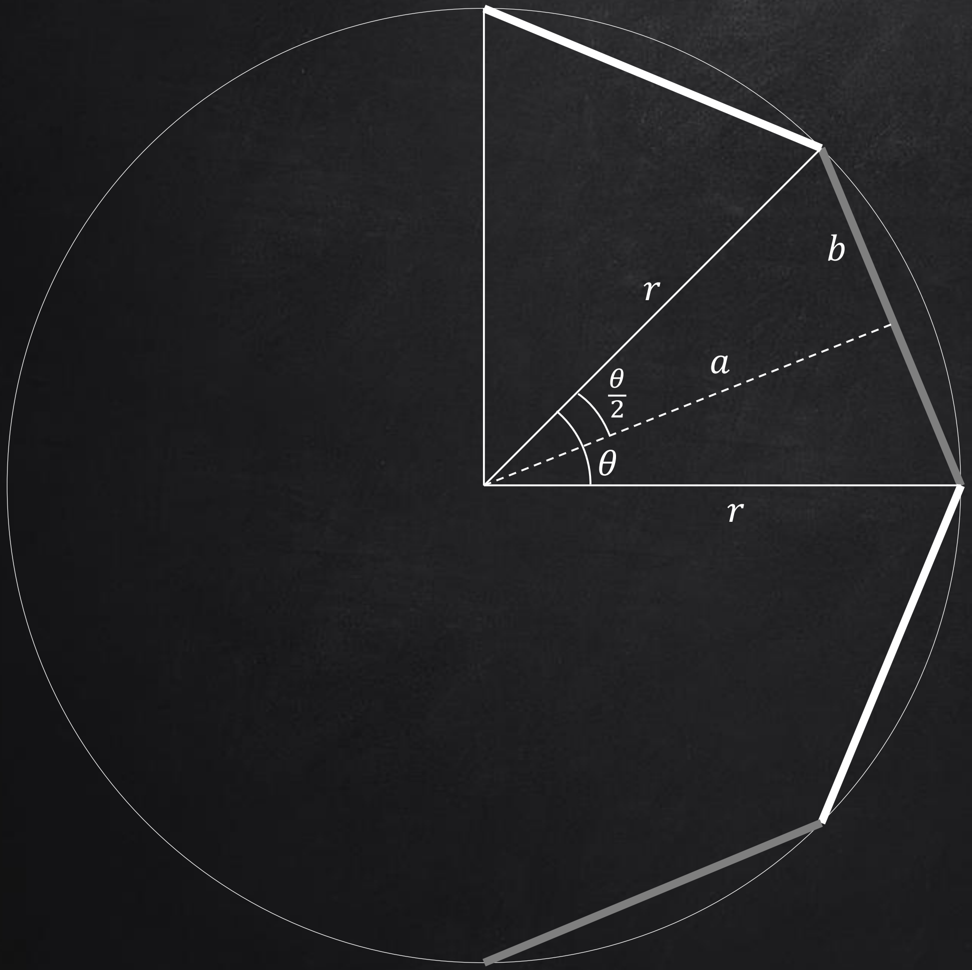
Not so fast!

- ✗ Irregular quadrilaterals generate irregular tetrahedra.
- ✗ Overlapping will occur.
- ✗ Pieces not necessarily align to features.
- ✗ Piece convexity is not guaranteed*.
- ✗ Is the structure stable?

* From tiles with more than 4 sides.

Our Method
(For cylinder only)

- ✓ Polygonal approximation of the cylinder using **squares**.
- ✓ Black squares produce **regular tetrahedra**.
- ✓ White squares produce **quasi-tetrahedra**.
- ✓ Parameters: **radius** of the cylinder, **# rings, # pieces per ring**.



r: radius of the cylinder

m: #pieces per ring

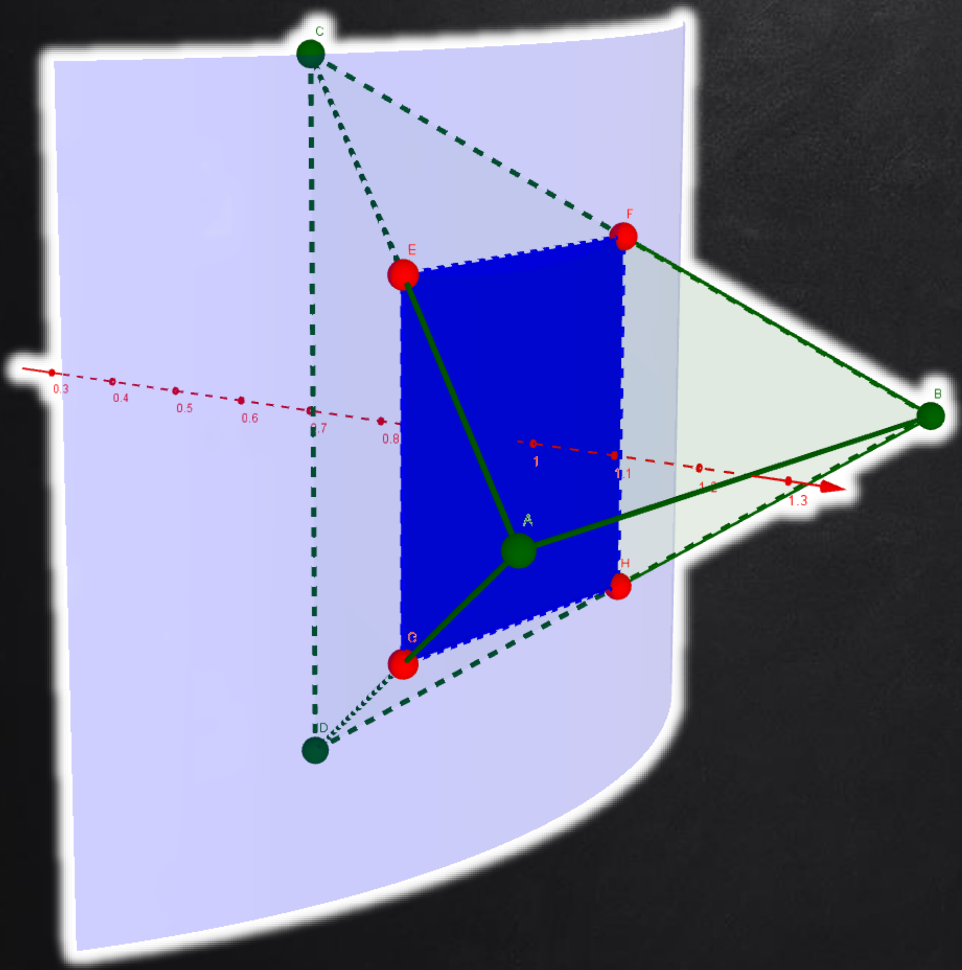
n: #rings

$$\theta = \frac{2\pi}{m} \Rightarrow \frac{\theta}{2} = \frac{\pi}{m}$$

$$\underbrace{\frac{a}{r} = \cos\left(\frac{\theta}{2}\right) \Rightarrow a = r\cos\left(\frac{\pi}{m}\right)}_{\text{Apothem}} \qquad \underbrace{\frac{b}{r} = \sin\left(\frac{\theta}{2}\right) \Rightarrow b = r\sin\left(\frac{\pi}{m}\right)}_{\text{Half-length of the square}}$$

$$l = 2b \Big\}\ \text{Length of the square}$$

$$\underbrace{h = nl = 2nr\sin\left(\frac{\pi}{m}\right)}_{\text{Height of the cylinder}}$$



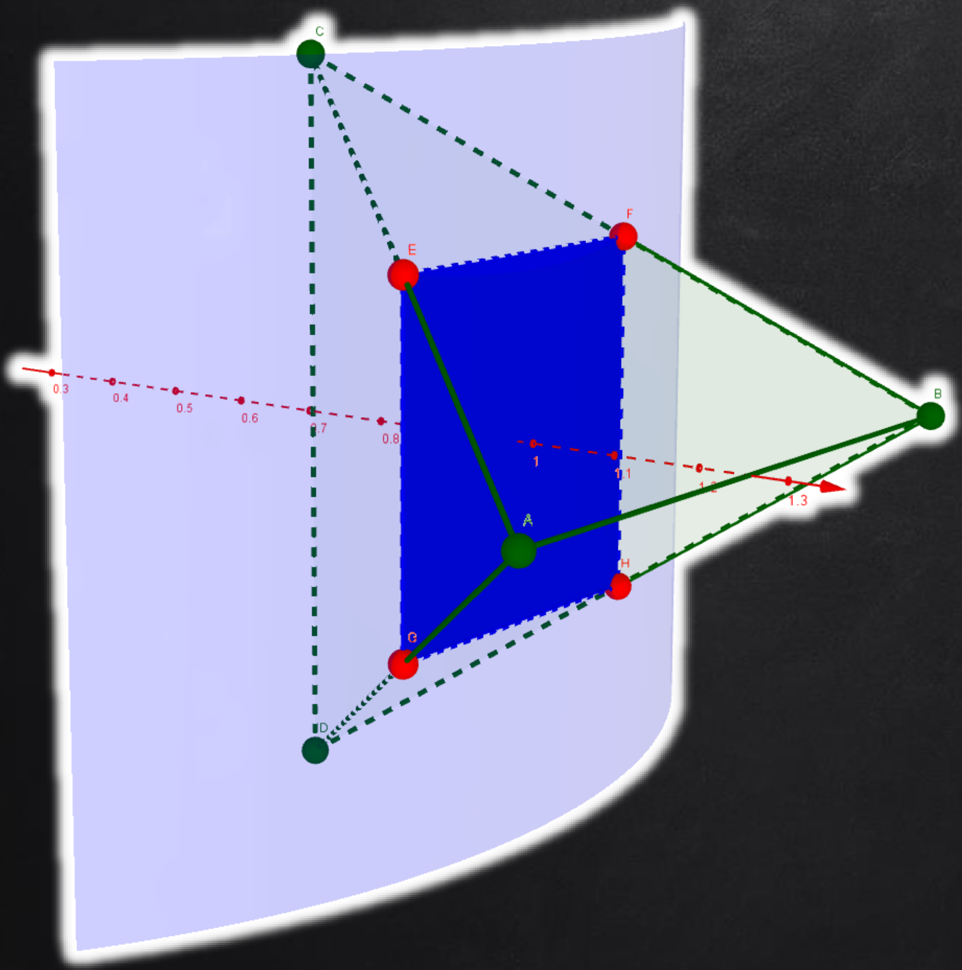
Assume vertices A, B, C, D define a regular tetrahedron. Segments $\overline{AB}$ and $\overline{CD}$ correspond to its horizontal and vertical segments respectively.

The length of the mid section of the tetrahedron (square) is l. Then, the length of all segments of the tetrahedron is $2l$.

The vertices of the tetrahedron are defined as:

$$\begin{aligned} A &= (a+k, 0, l) \\ B &= (a+k, 0, -l) \\ C &= (a-k, l, 0) \\ D &= (a-k, -l, 0) \end{aligned}$$

Where a is the apothem of the polygonal approximation of the cylinder, and k is the distance from the square to both horizontal and vertical segments of the tetrahedron.



Then:

$$\|AC\| = 2l$$

$$\sqrt{\big((a-k)-(a+k)\big)^2 + (l-0)^2 + (0-l)^2} = 2l$$

$$\sqrt{(a-k-a-k)^2 + l^2 + (-l)^2} = 2l$$

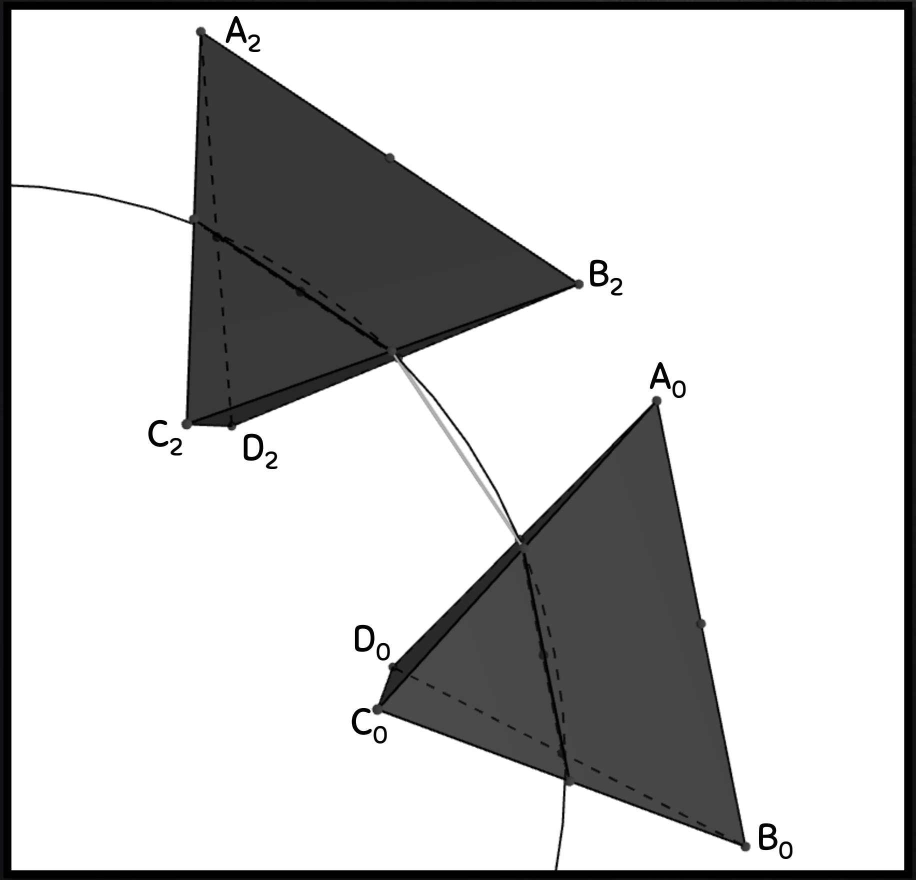
$$\sqrt{4k^2 + l^2 + l^2} = 2l$$

$$4k^2 + 2l^2 = 4l^2$$

$$4k^2 = 2l^2$$

$$2k = l\sqrt{2}$$

$$\boxed{k = \frac{l\sqrt{2}}{2} = r\sin\left(\frac{\pi}{m}\right)\sqrt{2}}$$

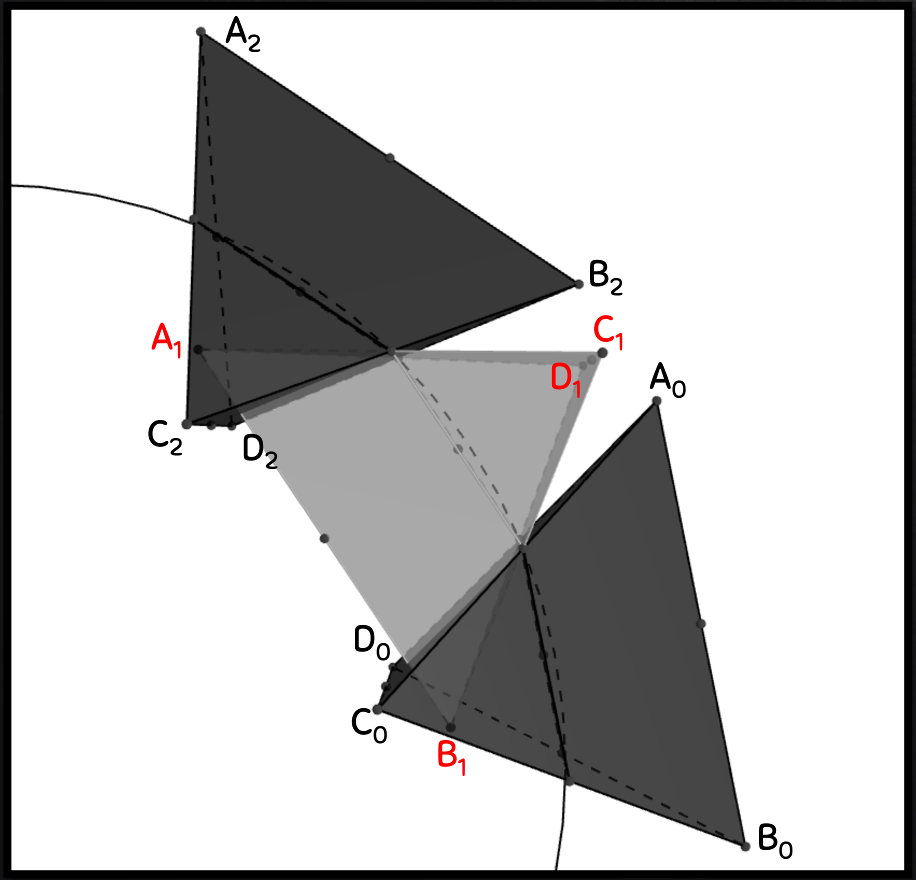


Generate the black regular tetrahedra:

$$Center_i = (a\cos\theta_i\,, h_i, a\sin\theta_i)$$
$$N_i = \|Center_i\|$$
$$Front_i = Center_i + (N_i.x, 0, N_i.z)$$
$$Back_i = Center_i - (N_i.x, 0, N_i.z)$$

$$A_i = (Front_i + l \cdot \|(-\sin\theta_i, 0, \cos\theta_i)\|)$$
$$B_i = (Front_i - l \cdot \|(-\sin\theta_i, 0, \cos\theta_i)\|)$$
$$C_i = \left(Back_i + (0, l, 0)\right)$$
$$D_i = \left(Back_i - (0, l, 0)\right)$$

$$\forall\ i = 0, 2, 4 \ldots\ i < m$$



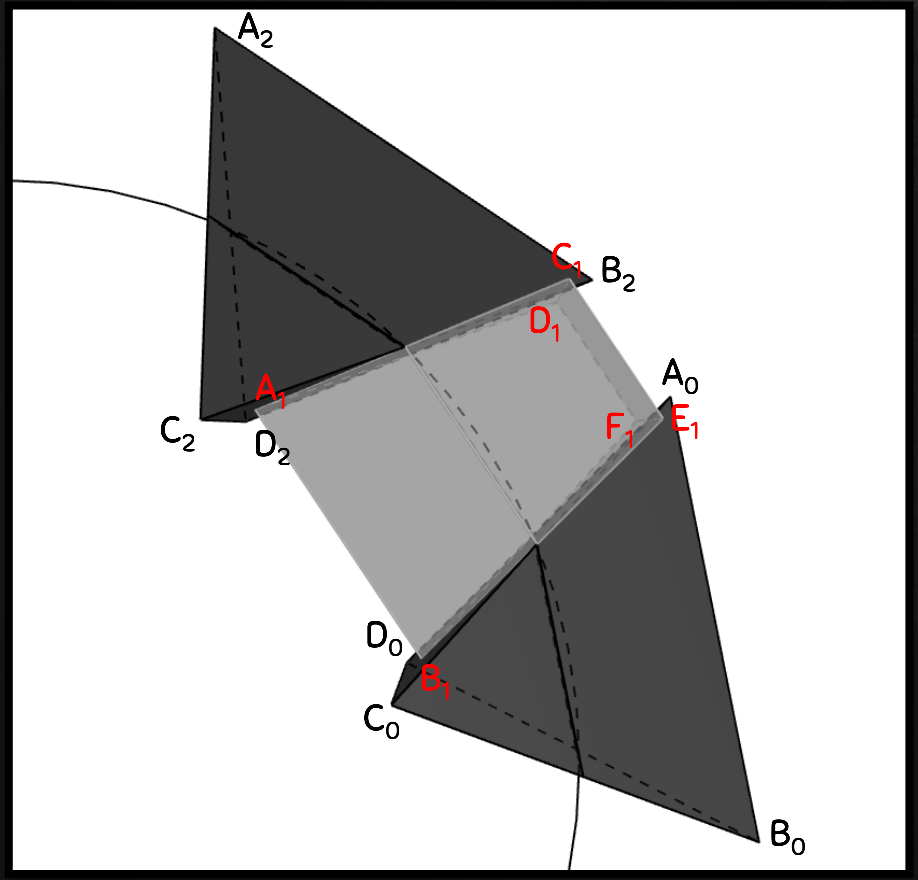
Generate the white regular tetrahedra:

$$Center_i = (a\cos\theta_i\,, h_i, a\sin\theta_i)$$
$$N_i = \|Center_i\|$$
$$Front_i = Center_i + (N_i.x, 0, N_i.z)$$
$$Back_i = Center_i - (N_i.x, 0, N_i.z)$$

$$A_i = (Back_i + l \cdot \|(-\sin\theta_i, 0, \cos\theta_i)\|)$$
$$B_i = (Back_i - l \cdot \|(-\sin\theta_i, 0, \cos\theta_i)\|)$$
$$C_i = \left(Front_i + (0, l, 0)\right)$$
$$D_i = \left(Front_i - (0, l, 0)\right)$$

$$\forall\, i = 1, 3, 5 \ldots\ i < m$$

Generate the quasi-tetrahedra:

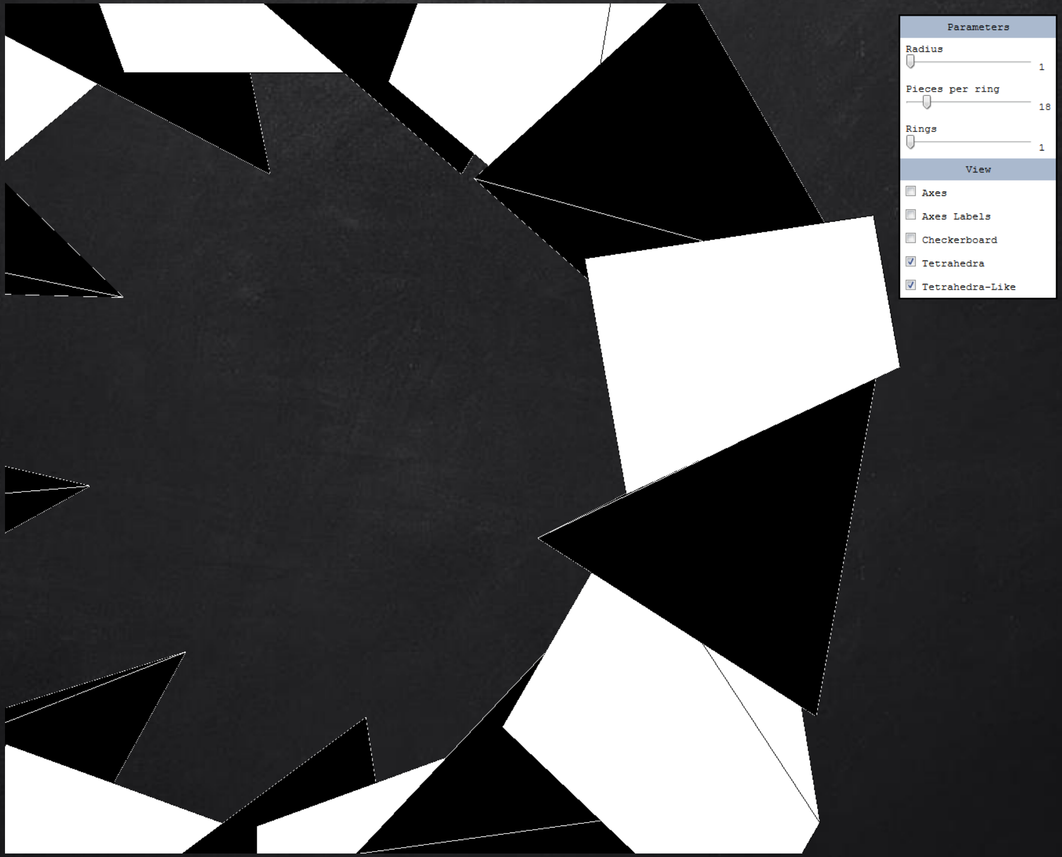


$$R_1 = \mathrm{ray}(A_i, B_i - A_i)$$
$$R_2 = \mathrm{ray}(B_i, A_i - B_i)$$
$$R_3 = \mathrm{ray}\left(\frac{C_i + D_i}{2}, \|(-\sin\theta_i, 0, \cos\theta_i)\|\right)$$
$$R_4 = \mathrm{ray}\left(\frac{C_i + D_i}{2}, \|(\sin\theta_i, 0, -\cos\theta_i)\|\right)$$
$$P_1 = Intersect(R_3, B_{i+1}, D_{i+1}, C_{i+1})$$
$$P_2 = Intersect(R_4, A_{i-1}, C_{i-1}, D_{i-1})$$

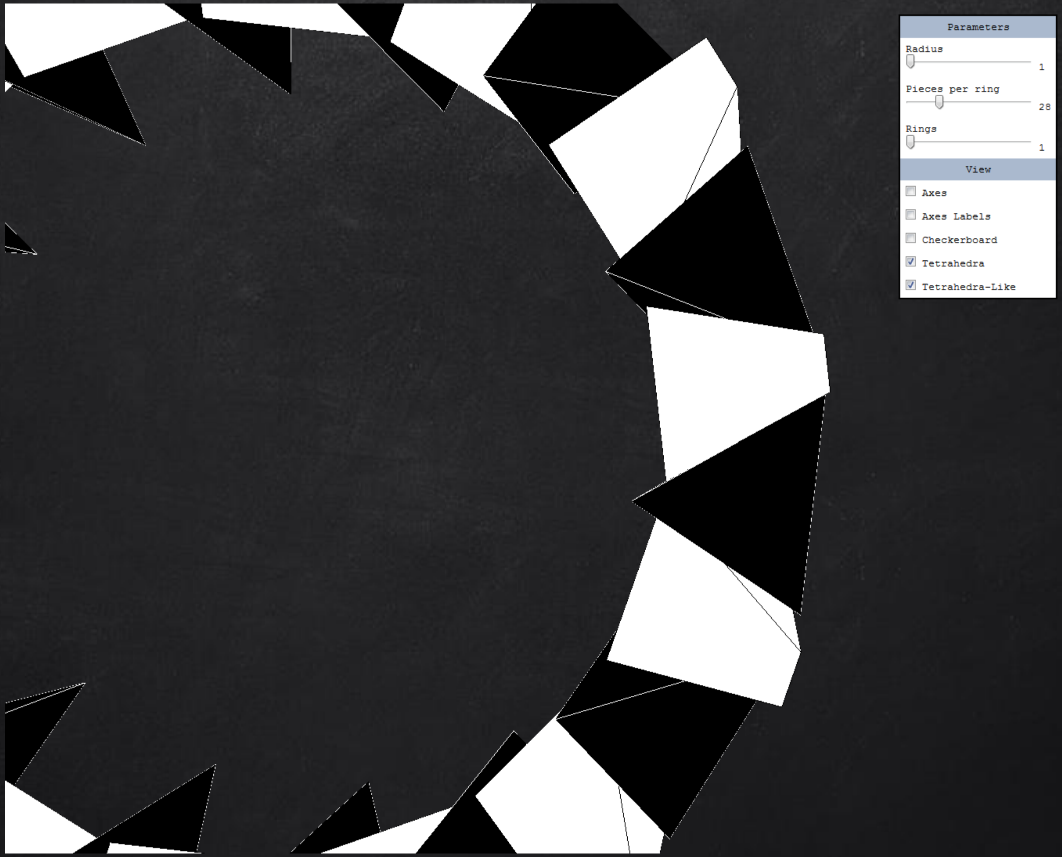
$$A_i = Intersect(R_1, B_{i+1}, C_{i+1}, D_{i+1})$$
$$B_i = Intersect(R_2, A_{i-1}, D_{i-1}, C_{i-1})$$
$$C_i = (P_1.x, C_i.y, P_1.z)$$
$$D_i = (P_1.x, D_i.y, P_1.z)$$
$$E_i = (P_2.x, C_i.y, P_2.z)$$
$$F_i = (P_2.x, D_i.y, P_2.z)$$

$$\forall\, i = 1, 3, 5 \ldots\ i < m$$

The Intersect function is the ray-triangle intersection.



Quasi-tetrahedra resembles tetrahedra as $m \to \infty$



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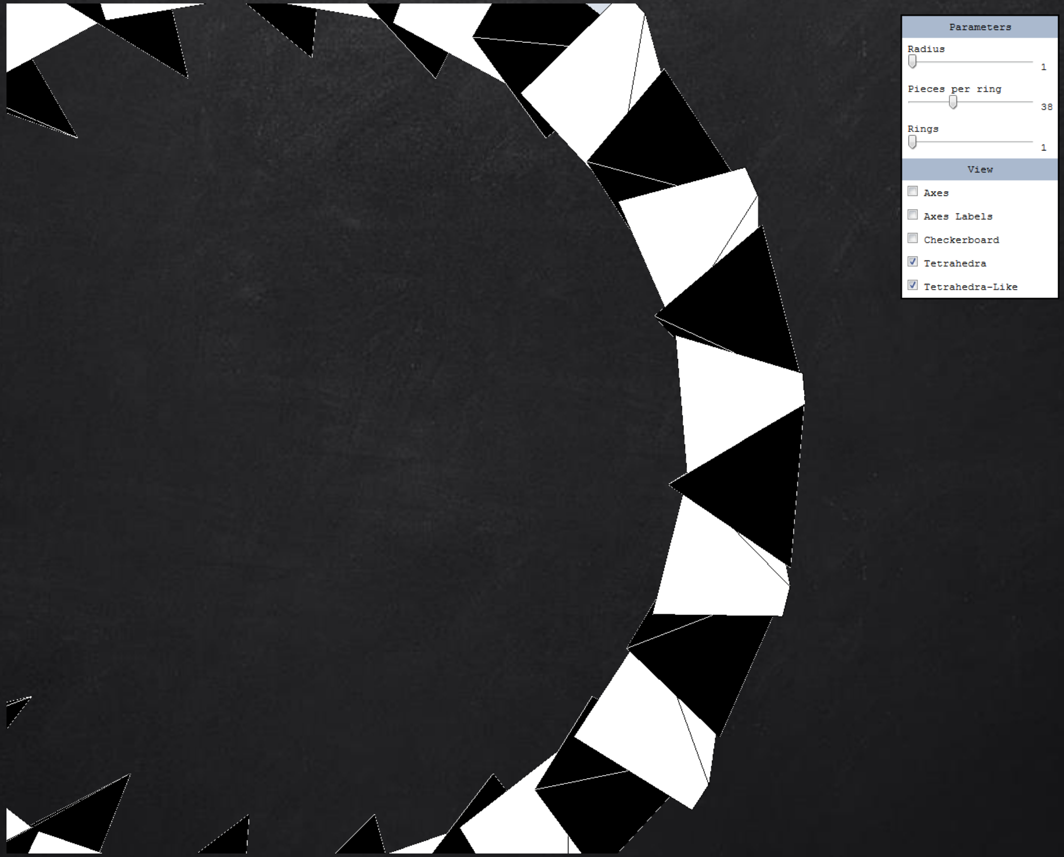

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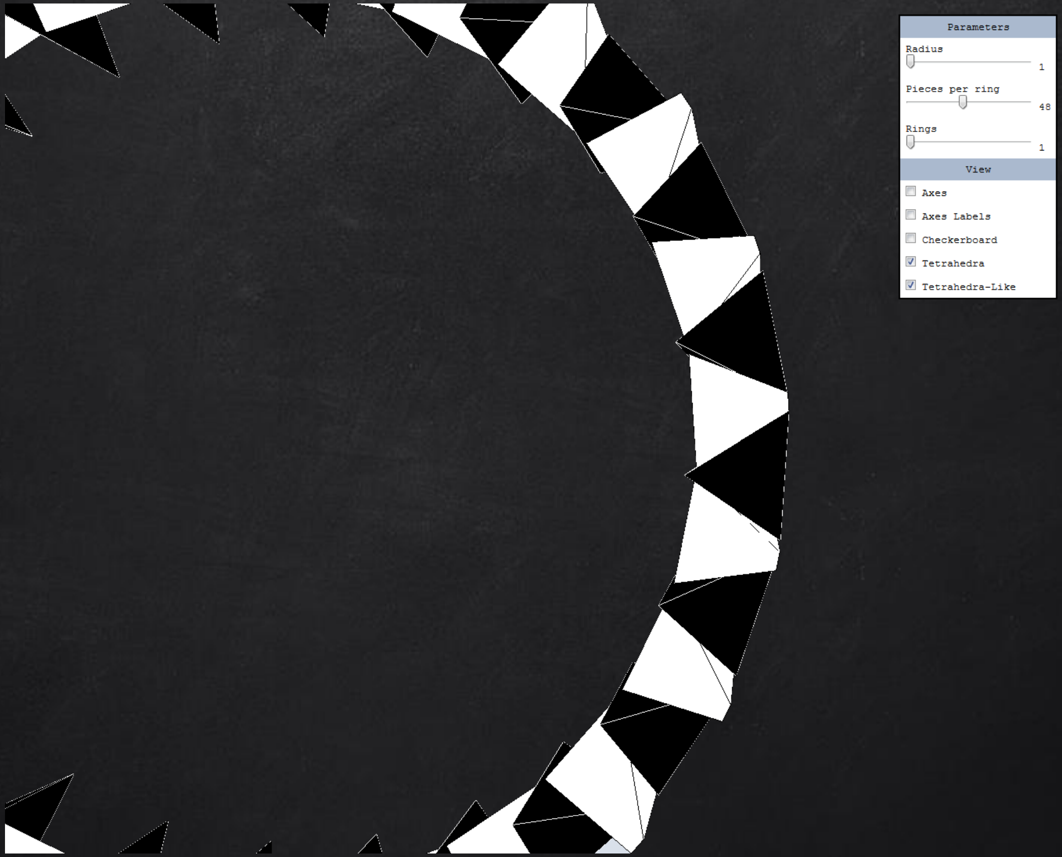

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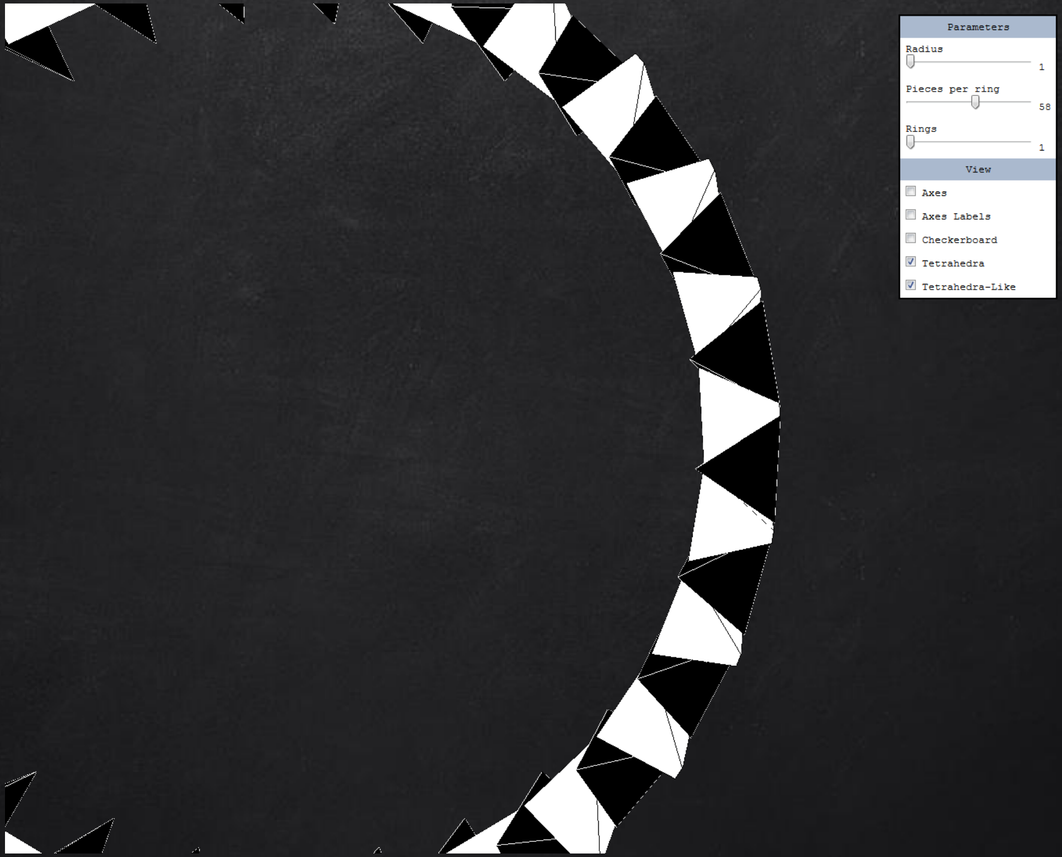

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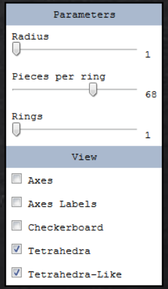
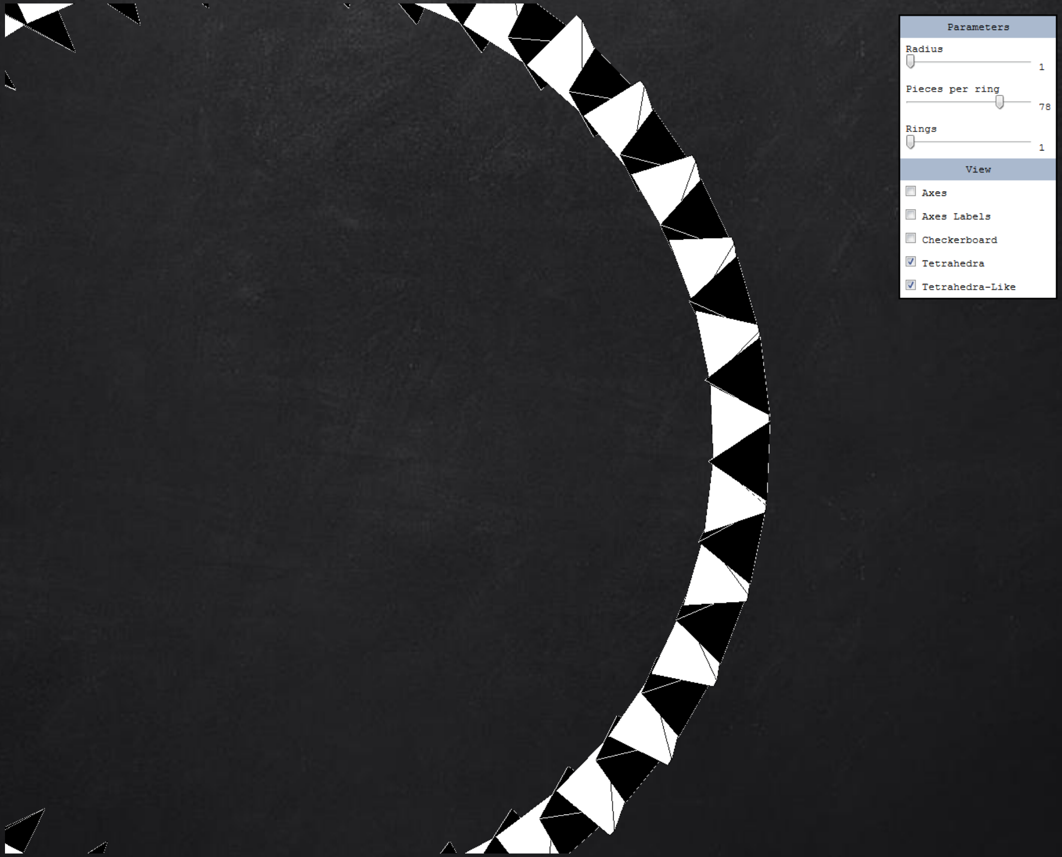


Quasi-tetrahedra resembles tetrahedra as $m \to \infty$



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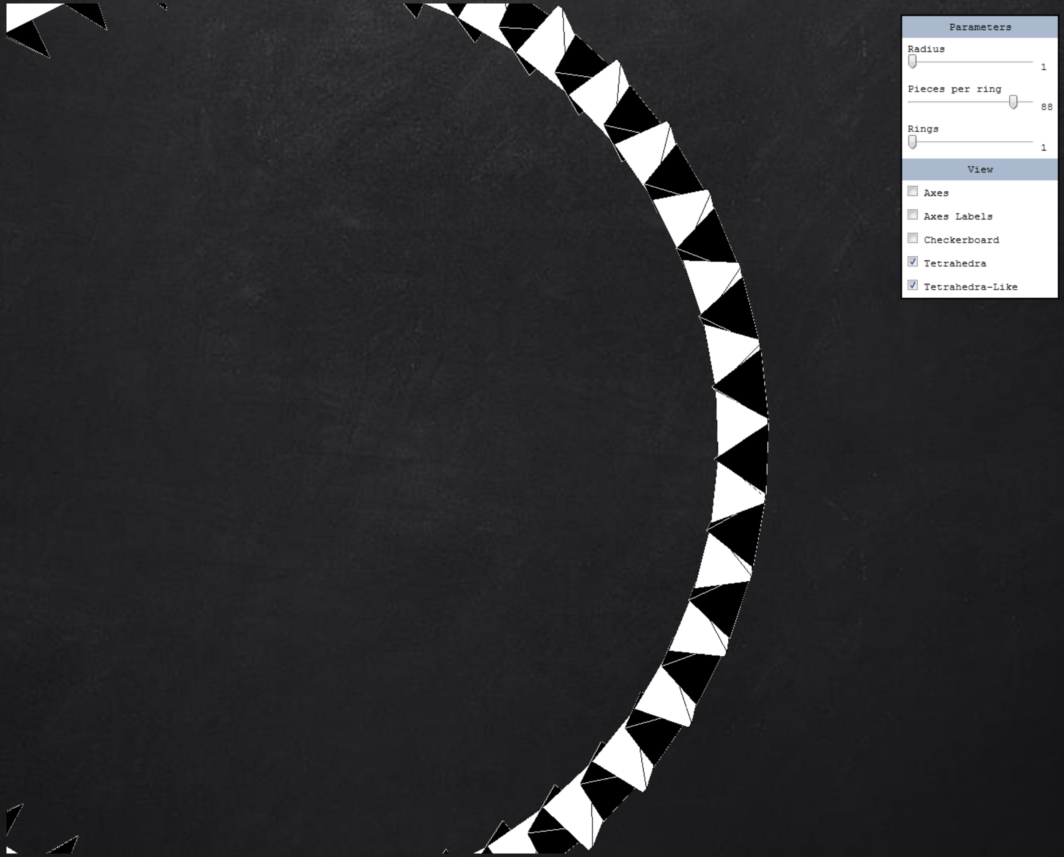

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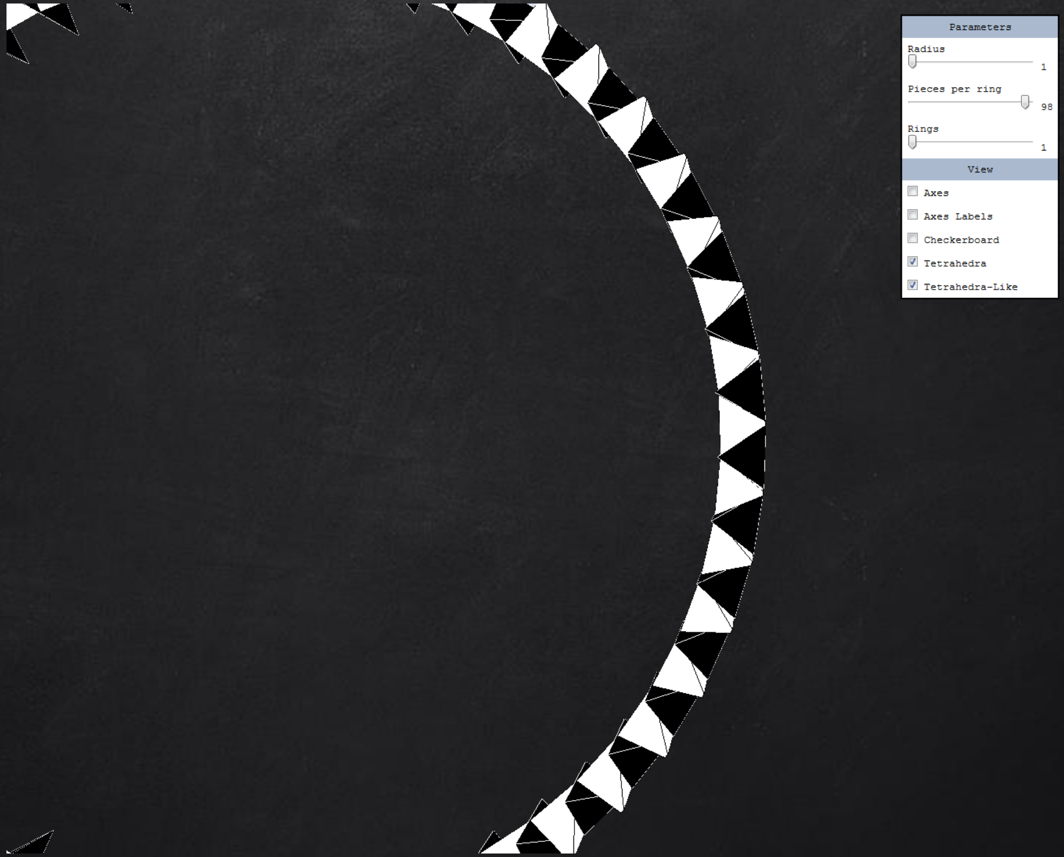

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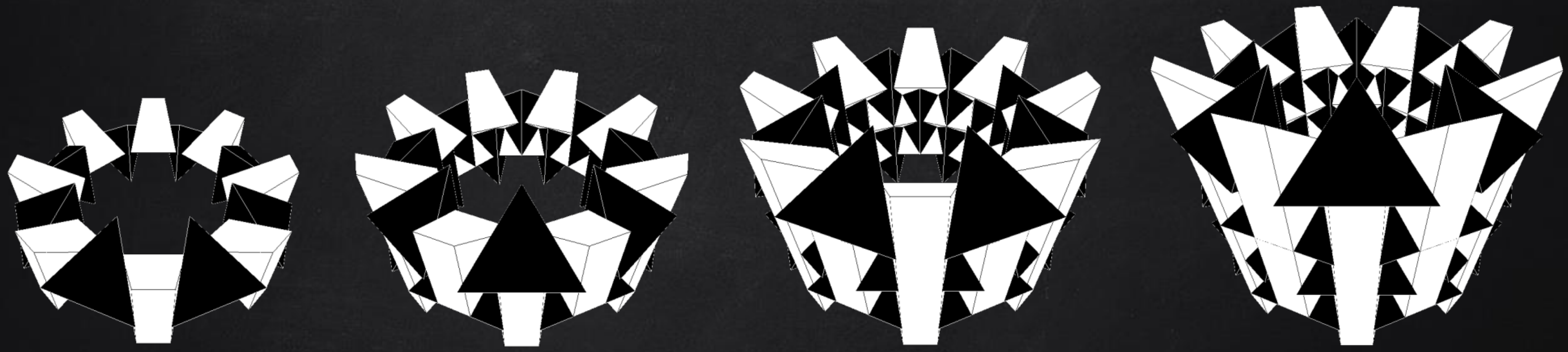
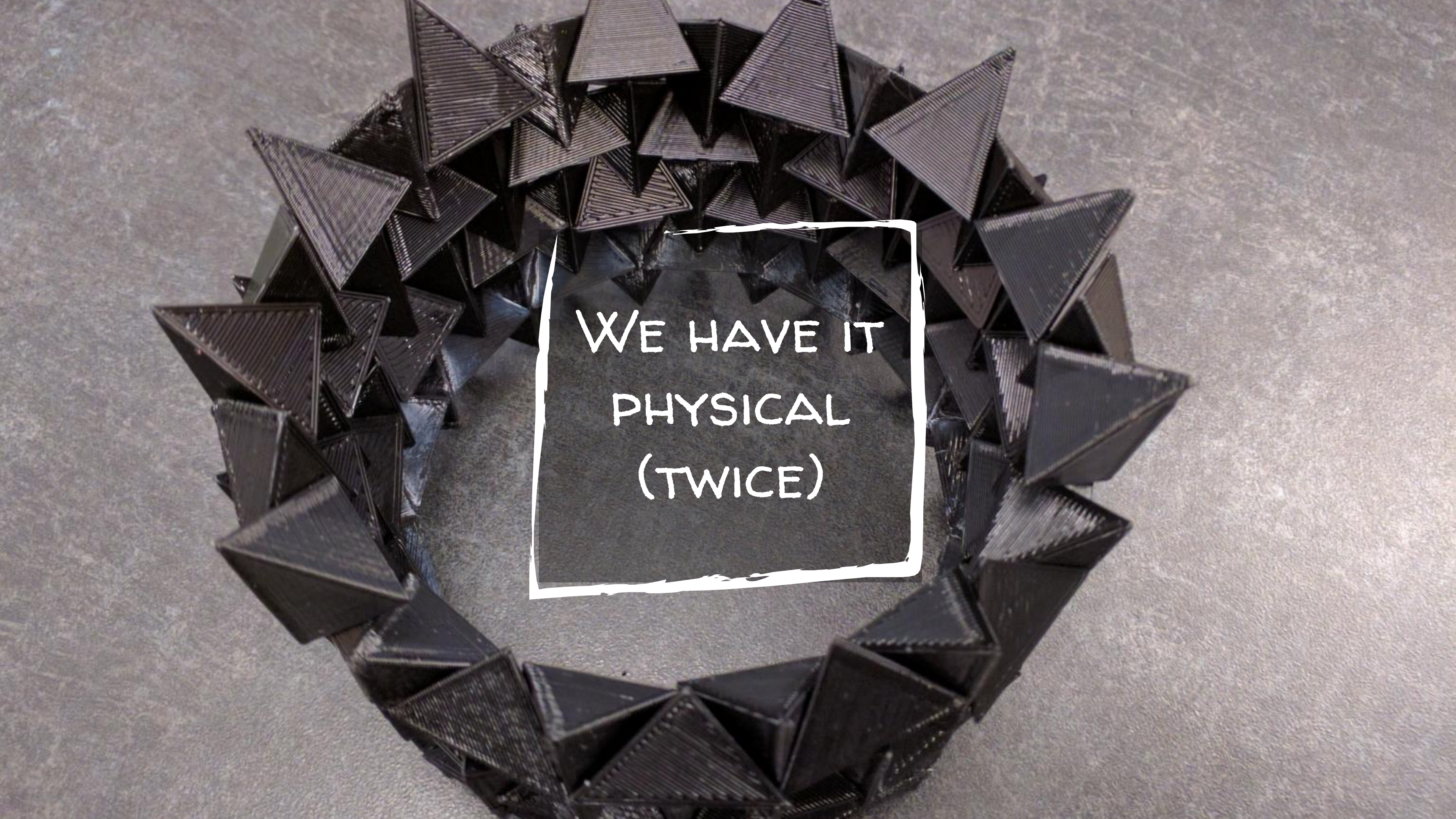
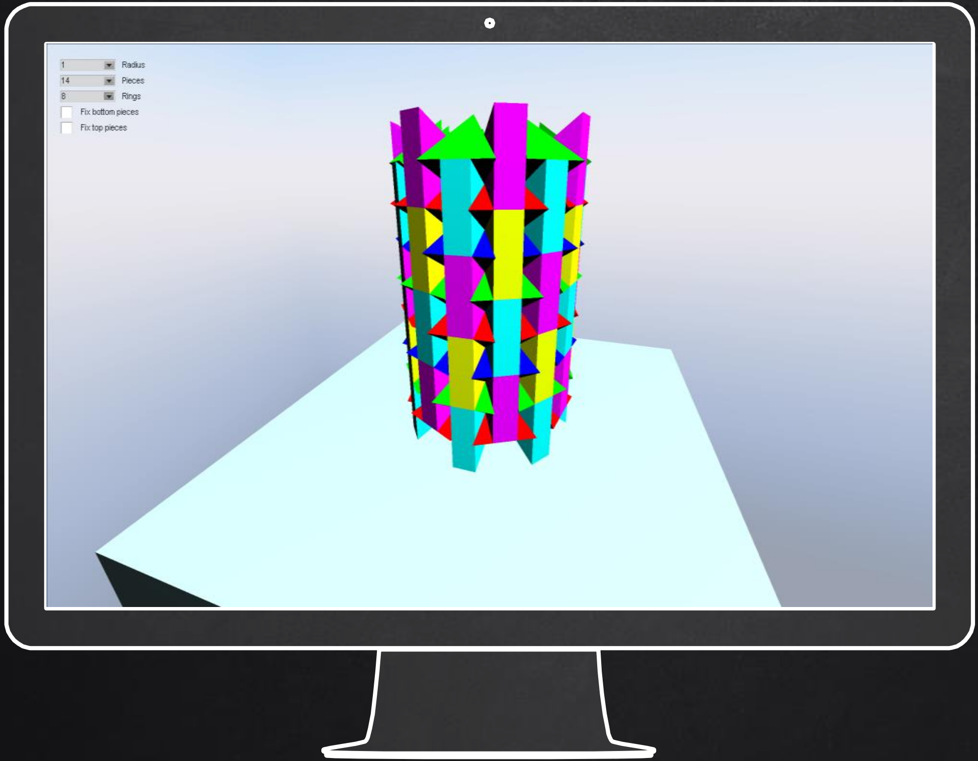


For each ring rotate θ_i by $\frac{\theta}{2}$


We have it
physical
(twice)



We have it on simulation

TICYL – Topological Interlocking Cylinder

Conclusions

- It is possible to build TICs based on a cylinder.
- Structure requires caps at both endpoints.
- Generation method is not generalizable from proposed methods under regular conditions (from previous talks).
- Could it be stable?

THANKS!

Any questions?

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Credits

Special thanks to all the people who made and released these awesome resources for free:

- Presentation template by SlidesCarnival
- Photographs by Unsplash